Efficient Rare-event Simulation II: State-dependent Techniques

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In this second lecture on rare-event simulation we revisit some of the applications which motivated our discussion in the first lecture, namely, queueing networks, insurance models and Gaussian processes with complex dependence. The first lecture introduced the model of computational complexity that is used to evaluate the performance of rare-event simulation algorithms, it discussed based techniques for the design of efficient Monte Carlo algorithms and it finished with examples showing that in the types of motivating applications mentioned above the basic techniques typically will not yield efficient simulation algorithms.

In the second lecture we shall introduce recent techniques that can be applied to design efficient rare-event Monte Carlo algorithms for complex systems and heavy-tailed processes. These techniques are based on state-dependent importance sampling and conditional sampling ideas. In the context of light-tailed systems the methods can be connected, by means of a control theoretic formulation, to differential games. This approach was introduced by Dupuis and Wang in a series of papers in the last five years and it involves finding a subsolution to a so-called Isaacs equation. Such subsolution is typically found analyzing a coarse description of the most likely path to the rare event.

We will discuss how this approach is related to the study of stochastic Lyapunov bounds for Markov chains. The use of Lyapunov inequalities coupled with the construction of subsolutions allows to obtain sharper running times for the associated state-dependent algorithms. In addition, Lyapunov inequalities can also be applied to heavy-tailed settings by studying a coarse description of the most likely path that leads to the rare event of interest. In this sense, when applied to heavy-tailed estimation problems, Lyapunov methods provide a vehicle that parallels the use of subsolutions for light-tailed processes.

We plan to conclude the lecture by discussing other types of algorithms (other than state-dependent importance sampling) and some open problems. This lecture is based on the following papers.

Blanchet, J., Glynn, P. and Leder, K. (2009) On Lyapunov Inequalities and Subsolutions for Efficient Importance Sampling. *Preprint.*

Blanchet, J., Glynn, P. and Liu, J. C. (2007) Fluid heuristics, Lyapunov bounds and efficient importance sampling for a heavy-tailed G/G/1 queue. *QUESTA.* 57, 99-113.

Blanchet, J. and Glynn, P. (2008) Efficient rare-event simulation for the maximum of a heavy-tailed random walk. *Ann. of Appl. Probab.* 18, 1351-1378.

Dupuis, P. and Wang, H. (2004) Importance sampling, large deviations, and differential games. *Stoch. and Stoch. Reports.* 76, 481-508.

Dupuis, P. and Wang, H. (2007) Subsolutions of an Isaacs equation and efficient schemes of importance sampling. *Mathematics of Operations Research.* 32, 723-757.