Oblivious Interference Scheduling

Alexander Fanghänel[1] Thomas Keßelheim[1]
Harald Räcke[2] Berthold Vöcking[1]

[1]Department of Computer Science
RWTH Aachen, Germany

[2]Department of Computer Science
University of Warwick, U. K.

Lunteren, January 2009

Motivation

Signals sent by different sources in multipoint radio networks need to be coordinated because they interfere.

The Media Access Control (MAC) layer ...

... provides single-hop full-duplex communication channels in multipoint networks to higher layers of the protocol stack.

We study the scheduling problems arising on the MAC layer from an algorithmic point of view.

Informal problem statement

The interference scheduling problem

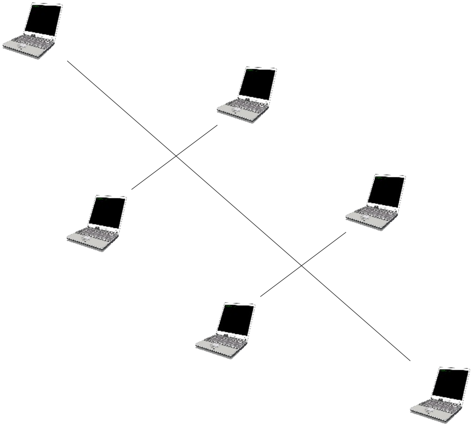
Given n pairs of points $(u_1, v_1), \ldots, (u_n, v_n)$ from a metric space, assign

- power levels $p_1, \ldots, p_n > 0$ and
- colors $c_1, \ldots, c_n$ from $\{1, \ldots, k\}$

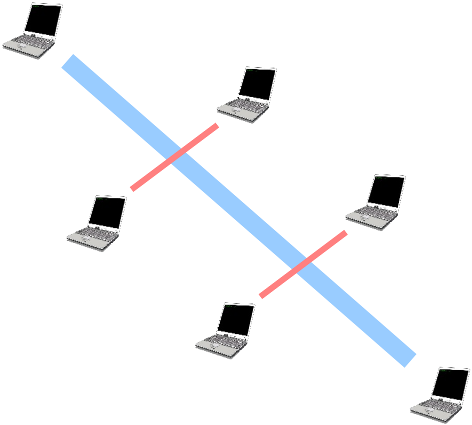
such the pairs in each color class "can communicate simultaneously" at the given power levels.

Objective: Minimize the number of colors k.

Illustration



Illustration



Modelling aspects

Graph-based vicinity models

- Two nodes in the radio network are connected by an edge in a communication graph if and only if they are in mutual transmission range.
- Interference is modelled through independence constraints: If a node u transmits a signal to an adjacent node v, then no other node in the k-hop neighborhood of u, for $k \geq 1$, can receive another signal.

The problem with this modelling approach is that it ignores that neither radio signals nor interference end abruptly at a boundary.

Modelling aspects

Let $\alpha \geq 1$ *(path loss exponent)* and $\beta > 0$ *(gain)* be fixed.

The physical model

- Let $\delta(u, v)$ denote the distance between the nodes u and v.
- The *loss* between u and v is defined as $\ell(u, v) = \delta(u, v)^\alpha$.
- A signal sent with power p by node u is received by node v at a strength of $p/\ell(u, v)$.
- SINR constraint: Node u can successfully decode this signal if its strength is larger than β times the sum of the strength of other simultaneously sent signals plus ambient noise ν.

SINR = signal to interference plus noise ratio
If not stated differently, we assume $\alpha = 2$, $\beta = 1$, $\nu = 0$.

Formal problem statement

The interference scheduling problem (directed variant)

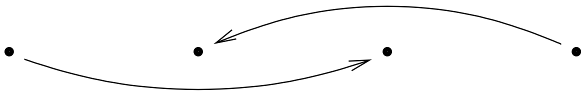
Given n pairs of points $(u_1, v_1), \ldots, (u_n, v_n)$ from a metric space, assign power levels $p_1, \ldots, p_n > 0$ colors $c_1, \ldots, c_n$ from $\{1, \ldots, k\}$ such that, for every $i \in [n] := \{1, \ldots, n\}$, it must hold the directed SINR constraint

$$\frac{p_i}{\ell(u_i, v_i)} > \beta \left(\sum_{\substack{j \in [n] \setminus \{i\} \\ c_j = c_i}} \frac{p_j}{\ell(u_j, v_i)} \right) .$$

Objective: Minimize the number of colors k.

[Moscibroda and Wattenhofer, INFOCOM 2006]

Example: overlapping pairs on a line



Overlapping Pairs cannot be scheduled simultaneously ...

Example: nested pairs on a line

Consider six equally spaced points on the line:

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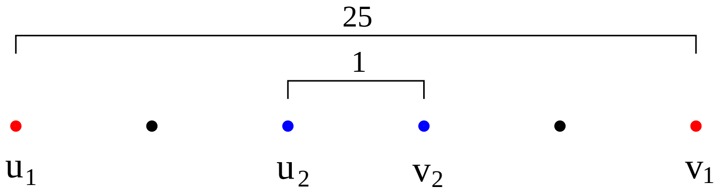
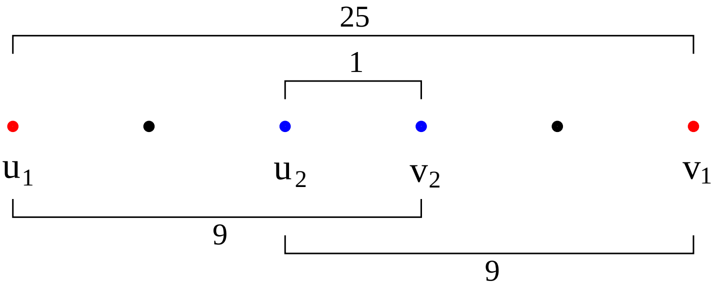


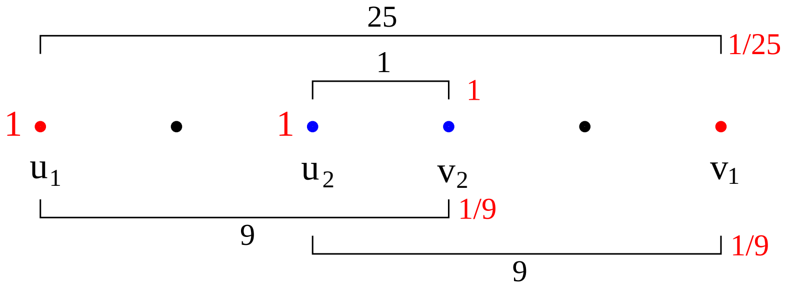
Example: nested pairs on a line

Consider six equally spaced points on the line:



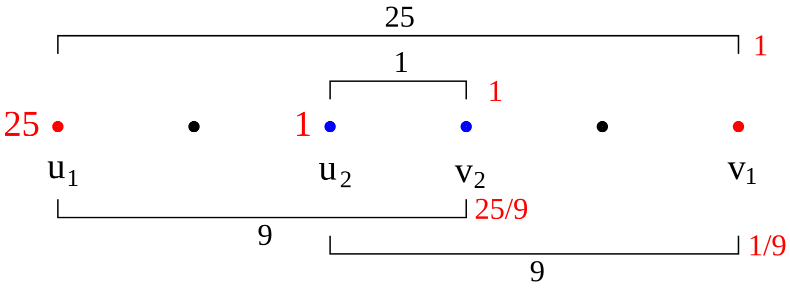
Example: nested pairs on a line

Uniform power assignment:



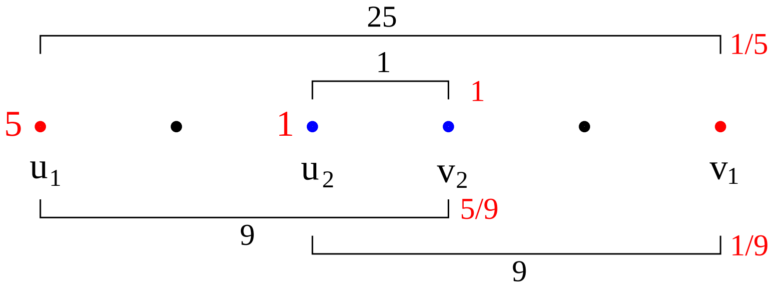
Example: nested pairs on a line

Linear power assignment:



Example: nested pairs on a line

Square root power assignment:



Oblivious Power Assignments

Definition:

A power assignment is called *oblivious* if there is a function $f : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ such that, for every $i \in [n]$, $p_i = f(\ell(u_i, v_i))$.

Examples: uniform, linear, square root

Advantage: Easy to implement (in a distributed fashion)

Question: Is there a universally good oblivious power assignment, i.e., a power assignment for which there exists a coloring using an almost optimal number of colors for every set of request pairs?

Related work

- Moscibroda and Wattenhofer, 2006, give an efficient algorithm for achieving strong connectivity among n points in Euclidean space with $O(\log^4 n)$ colors using a non-oblivious power assignment.
- Chafekar et al., 2007, show that for the linear power assignment there exists a coloring with only

$$O(\mathrm{opt}' \cdot \mathrm{polylog}(n, \Delta, \Gamma))$$

colors for any n request pairs in Euclidean space, where Δ denotes the aspect ratio, Γ the available power range, and opt' the optimal number of colors under a slightly more restrictive power range.

New result for directed SINR constraints

Question: Is there an oblivious power assignment with approximation factor polylog(n)?

Answer: – **NO!**

Theorem:

Let $f : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ be any oblivious power assignment function. There exists a family of instances on a line requiring $\Omega(n)$ colors under f but only $O(1)$ colors under a different power assignment.

Sketch of proof

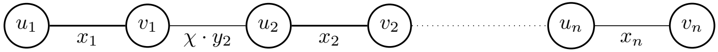
We distinguish three cases depending on the asymptotic behaviour of f.

1. f is asymptotically unbounded, that is, for every $c > 0$ and every $x_0 > 0$ there exists a value $x > x_0$ with $f(x) > c$
2. f is asymptotically bounded from above by some value $c > 0$ but does not converge to 0
3. f converges against 0

In this talk, we focus on the first case only.

Sketch of proof

We construct the following family of instances



χ is a sufficiently large constant (depending on β).

x_i and y_i depend on f and are defined as follows:

$$y_i = 2(x_{i-1} + y_{i-1}).$$

Given $x_1, \ldots, x_{i-1}$ and y_i, we choose x_i such that $x_i \geq y_i$ and

$$f(x_i) \geq y_i^{\alpha} \cdot \frac{f(x_j)}{x_j^{\alpha}} \qquad \text{for all } j < i.$$

This choice is always possible since f is asymptotically unbounded.

Sketch of proof

Let S be a set of pairs that can be scheduled simultaneously. Let $k = \min(S)$. As distances increase geometrically, $\delta(u_i, v_k) \leq 4\chi y_i$, for $i \in S \backslash \{k\}$.

The SINR constraint at receiver v_k yields

$$\beta \sum_{i \in S \backslash \{k\}} \frac{p_i}{\ell(u_i, v_k)} \leq \frac{p_k}{\ell(u_k, v_k)} = \frac{f(x_k)}{x_k^\alpha} \quad .$$

Combining these equations gives

$$\frac{1}{\beta} \frac{f(x_k)}{x_k^\alpha} \geq \sum_{i \in S \backslash \{k\}} \frac{p_i}{\ell(u_i, v_k)} \geq \sum_{i \in S \backslash \{k\}} \frac{y_i^\alpha \frac{f(x_k)}{x_k^\alpha}}{(4\chi y_i)^\alpha} = \frac{|S| - 1}{(4\chi)^\alpha} \cdot \frac{f(x_k)}{x_k^\alpha} \quad .$$

Thus $|S| \leq \frac{(4\chi)^\alpha}{\beta} + 1 = O(1)$.

Sketch of proof

Now consider the non-oblivious power assignment $p_i = \sqrt{2^i}$.

- Observe that distance increase geometrically as $y_i \leq x_i$ and $y_{i+1} \geq 2x_i$.
- For this reason, the sum of interferences for the lower as well as for the higher indices form geometric series.
- Thus a constant fraction of all pairs may share the same color.

Hence we have shown that f requires $\Omega(n)$ colors while there is a coloring for which $O(1)$ colors suffice. □

However ...

Network standards demand that the MAC layer provides single-hop full-duplex communication channels.

Therefore one should study bidirectional rather than directed communication channels.

Formal problem statement

The interference scheduling problem (bidirectional variant)

Given n pairs of points $(u_1, v_1), \ldots, (u_n, v_n)$ from a metric space, assign power levels $p_1, \ldots, p_n > 0$ colors $c_1, \ldots, c_n$ from $\{1, \ldots, k\}$ such that, for every $i \in [n] := \{1, \ldots, n\}$ and $w \in \{u_i, v_i\}$, it must hold the bidirectional SINR constraint

$$\frac{p_i}{\ell(u_i, v_i)} > \beta \left(\sum_{\substack{j \in [n] \setminus \{i\} \\ c_j = c_i}} \max\left\{ \frac{p_j}{\ell(u_j, w)}, \frac{p_j}{\ell(v_j, w)} \right\} \right) .$$

Objective: Minimize the number of colors k.

Oblivious power assignment for bidirectional SINR constraints

The square root power assignment $\bar{p}$ sets the power level for a pair (u, v) equal to $\sqrt{\ell(u, v)}$.

Theorem

For any set of n bidirectional communication requests, $\bar{p}$ admits a coloring with at most $\text{polylog}(n)$ *times the minimal number of colors.*

We prove this result by showing that there is a subset $S \subseteq [n]$ with $|S| \geq n/\text{polylog}(n)$ that is β-*feasible* for $\bar{p}$, i.e., satisfies the SINR constraint with gain β using only one color.

Analysis: high-level description

For our analysis, we solve the following relaxation.

Node-loss scheduling

One is given a set of nodes $u_i \in V$ each coming with a loss parameter ℓ_i. One needs to specify a β-*feasible* subset $U \subseteq V$ with power levels, i.e., for all $i \in U$, it holds

$$\frac{p_i}{\ell_i} > \beta \left(\sum_{j \in U \setminus \{i\}} \frac{p_j}{\ell(i,j)} \right) .$$

The square root power assignment $\bar{p}$ sets the power level for a node-loss pair (u, ℓ) equal to $\sqrt{\ell}$.

Analysis: high-level description

Lemma (to be shown)

Given any set $|V|$ of node-loss pairs that is β-feasible for any power assignment, there exists a β'-feasible subset $U \subseteq V$ for $\bar{p}$ with $|U| \geq \frac{4}{5}|V|$ and $\beta' = \beta^{2/3}/\operatorname{polylog}(n)$.

Going back from node-loss pairs to pairs of nodes (requests), it follows that there is a β'-feasible subset $S \subseteq [n]$ of requests with $|S| \geq \frac{3}{5}n$.

Using a randomized coloring procedure, one can sparsify S by a polylogarithmic factor and obtain a subset S' of size $n/\operatorname{polylog}(n)$ that is β-feasible.

Analysis: From general to tree metrices

Proposition (Fakcharoenphol, Rao, Talwar, 2003)

Given a finite metric space (V, δ) there exist $r = O(\log |V|)$ edge weighted treees $T_1, \ldots, T_r$ with nodes set V such that

- $\forall (u, v) \in V^2$, $\forall i \in \{1, \ldots, r\}$: $\delta(u, v) \geq \delta_{T_i}(u, v)$.
- $\forall (u, v) \in V^2$, $\exists i \in \{1, \ldots, r\}$: $\delta(u, v) \leq \delta_{T_i}(u, v) \cdot O(\log V)$.

Applying this result, it remains only to show

Claim

Given any set $|V|$ of node-loss pairs from a tree metric that is β-feasible for any power assignment, there exists a β'-feasible subset $U \subseteq V$ for $\bar{p}$ with $|U| \geq 1 - \frac{1}{5r}|V|$ and $\beta' = \beta^{2/3}/\operatorname{polylog}(n)$.

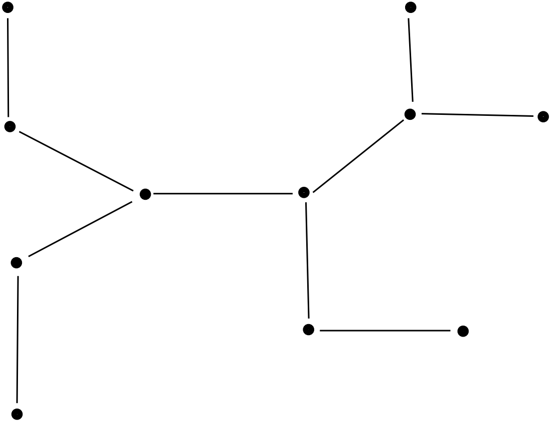
Analysis for star metrices

We show

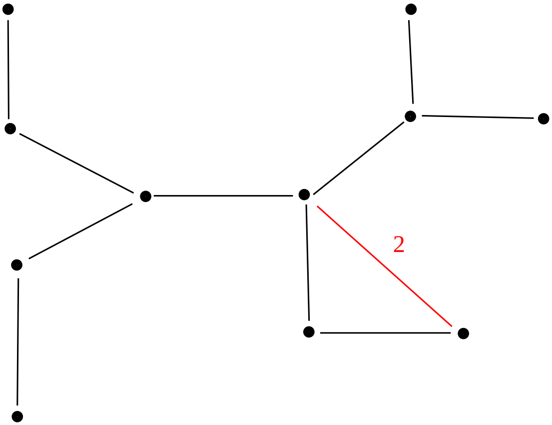
Lemma

Given any set $|V|$ of node-loss pairs from a star metric that is β-feasible for any power assignment, there exists a β'-feasible subset $U \subseteq V$ for $\bar{p}$ with $|U| \geq 1 - \frac{1}{5r}|V|$ and $\beta' = \beta^{2/3}/\operatorname{polylog}(n)$. □

Decomposing trees into stars



Decomposing trees into stars



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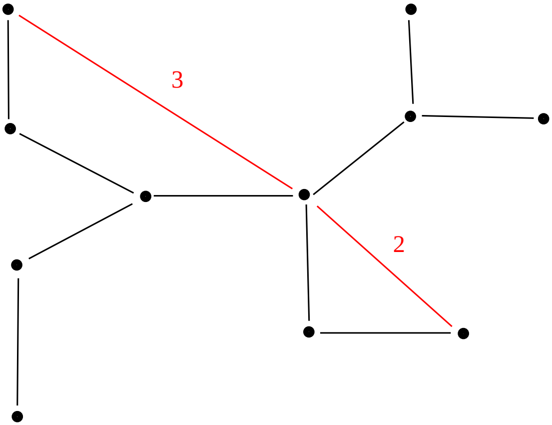

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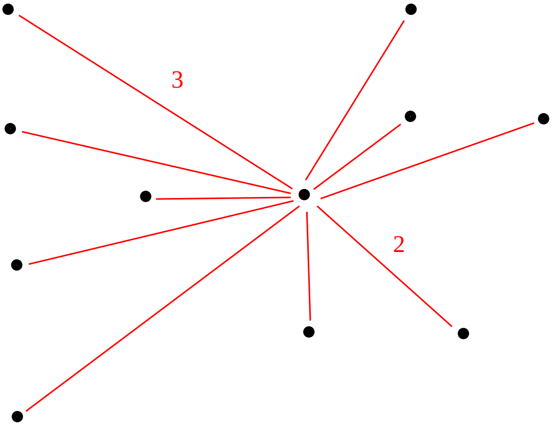

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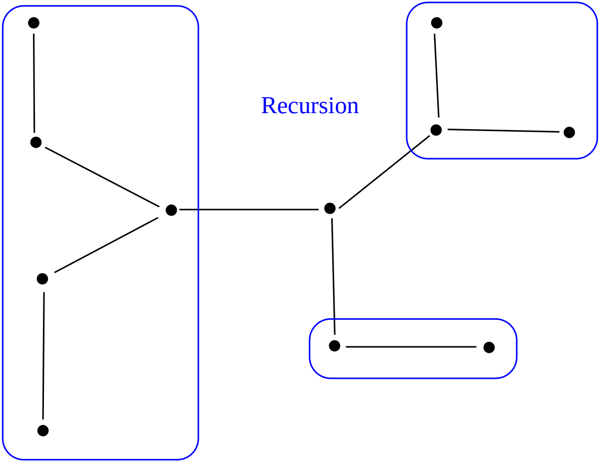

Algorithmic Aspects

- The presented analysis assumes that an optimal power assignment and coloring is known.
- Therefore, the given existence proof is not constructive.

Theorem

There is an efficient coloring algorithm for the square root power assignment that approximates the optimal number of colors up to a factor of $O(\log n)$. □

Conclusions & open problems

The square root power assignment has advantages against other assignments in some selected worst-case instances. What is the performance in random, perturbed or real world instances?

The schedule (coloring) for the square root assignment can be computed by a polynomial time algorithm. Is there a distributed scheduling policy that is suitable for application in practice?

Our analysis assumes that both communication partners of a pair use the same power. Can asymmetric assignments achieve a (significantly) better performance than symmetric ones?