# Strong Bounds on Perturbations Using Weak Derivatives

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We consider families of probability measures $\{\mu_\theta : \theta \in \Theta\}$, for some open, connected $\Theta \subset \mathbb{R}$, which are weakly differentiable, i.e., the mapping

$$\theta \mapsto \mathbb{E}_\theta[g(X)] = \int g(x)\mu_\theta(dx)$$

is differentiable, point-wise with respect to $g \in \mathcal{D}$, for a pre-defined class of cost-functions $\mathcal{D}$. Starting from the observation that weak differentiability implies implies strong (norm) Lipschitz continuity, where *strong* means *uniform with respect to* $g$, one can derive Lipschitz constants for various families of probability measures using the expression of the weak derivative.

In this talk we will explain how this method can be extended to product measures, in order to perform a sensitivity analysis for stochastic systems depending on a finite family of i.i.d. random variables. Eventually, we discuss possible extensions to infinite families of random variables which allow for studying strong stability of Markov chains. In particular, we illustrate our theory with an application to the waiting times in a $G/G/1$ queue.