ERASMUS RESEARCH INSTITUTE OF MANAGEMENT

Optimization Methods for Risk Management of Interest Rate Derivatives

Seminar
Mathematical Models for Financial Optimization
Lunteren, 20-Jan-2005

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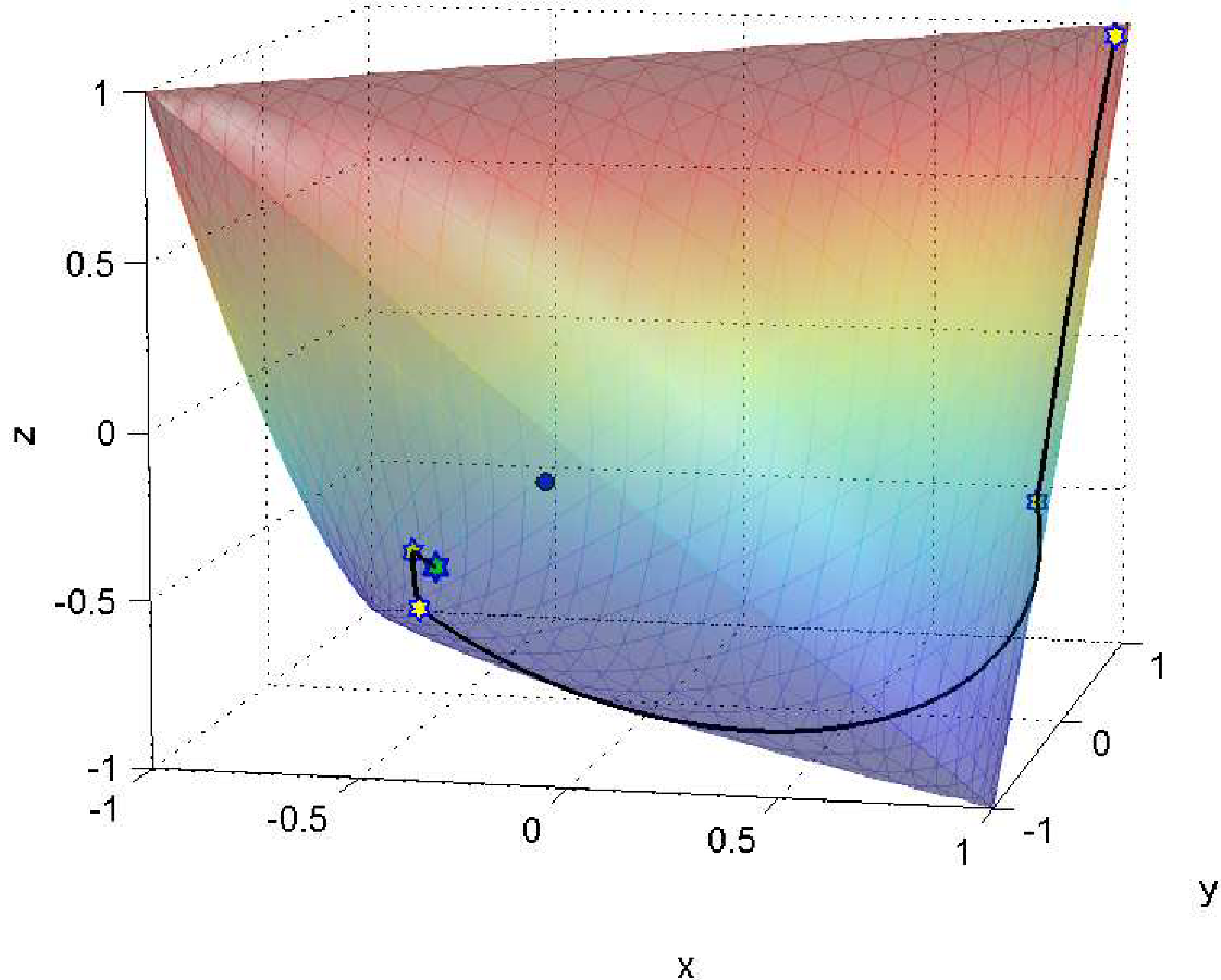
Outline

- Part 1:

- Risk managing interest rate derivatives:
- Model as tool to minimize variance of profit and loss (P&L)
- Compare performance of models in practice

- Conclusions:
- Some as expected, some controversial



z
1
0.5
0
-0.5
-1
x
-1
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0
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1
y
1
0
-1

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Part 1:

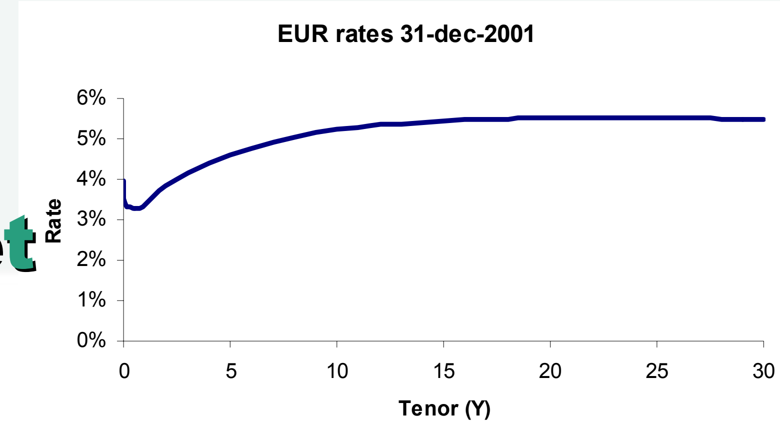
Comparison of model hedge performance in practice

Joint work with Antoon Pelsser

Outline of part 1

- Interest rate market
- Bermudan swaptions
- Model as interpolation tool
- Hedging
- Interest rate derivatives pricing models
- Results

Interest rate market

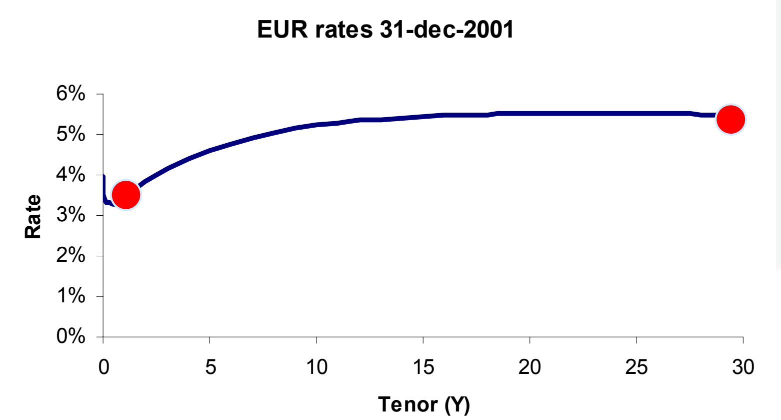


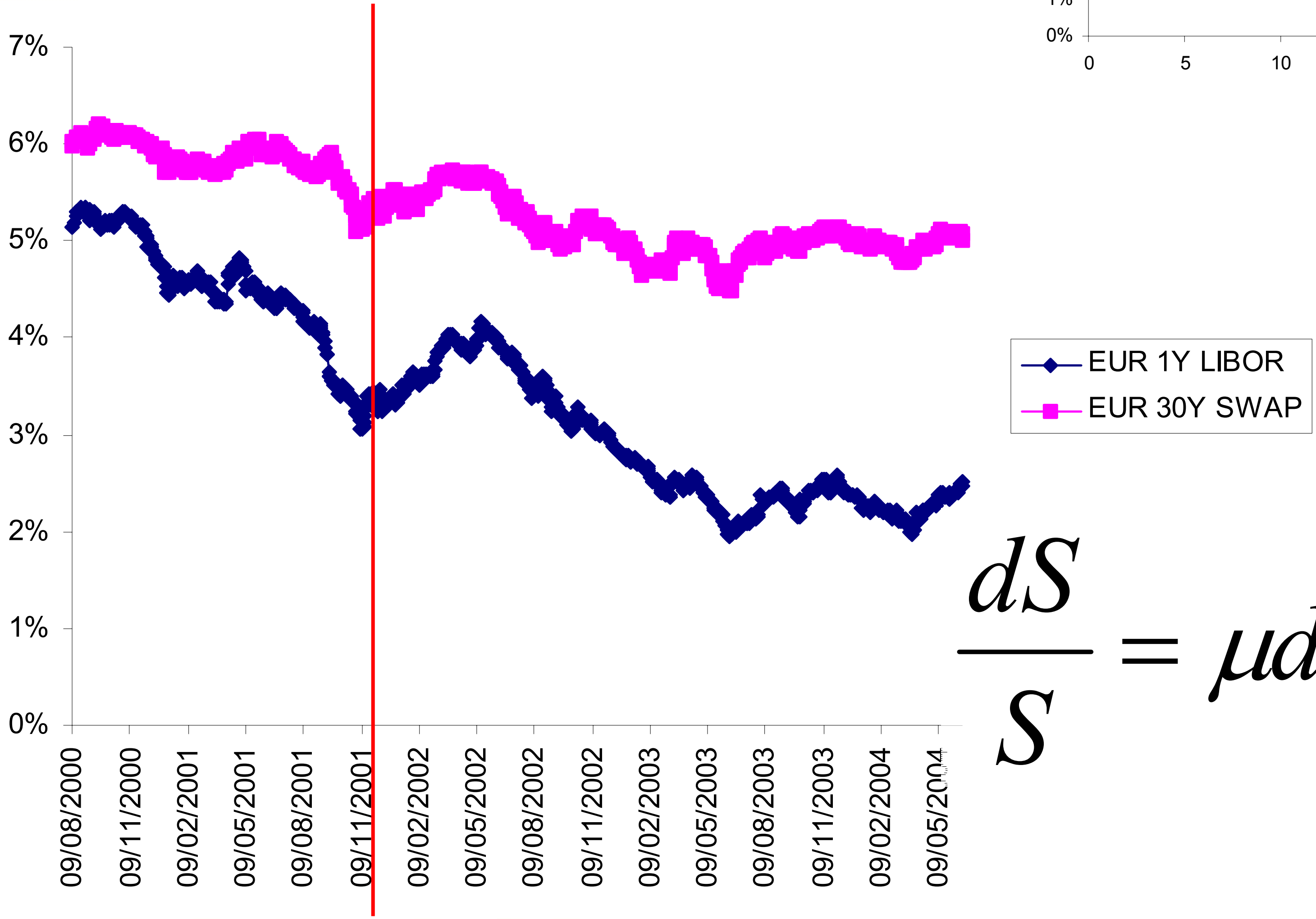
- Interest rate (IR) (swap rate)
- Borrowing/lending money over agreed period of time at agreed rate of interest

- IR may vary with length of deal (tenor)
- $\Rightarrow$ term structure (TS) of IR

- Forward borrowing/lending:
- Agree now to borrow starting from expiry over tenor period at agreed interest rate

Rates move over time



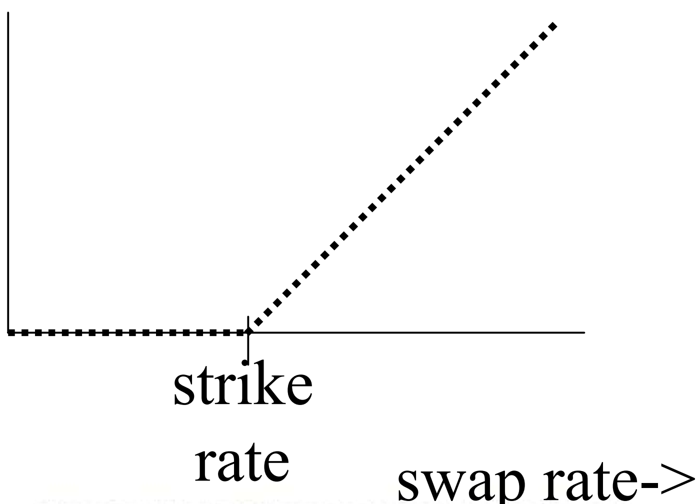


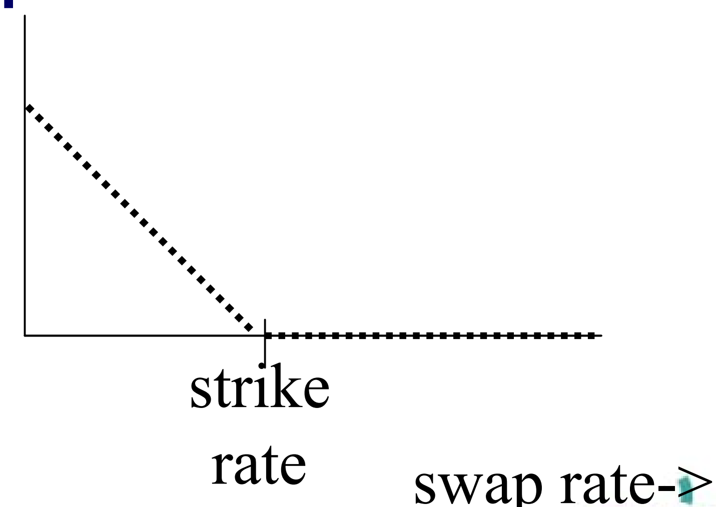
$$\frac{dS}{S} = \mu dt + \sigma dW$$

Interest rate options (swaptions)

- Holder has right to enter into borrowing/lending agreement at strike rate (European)
- Borrow ⇔ ‘Call on forward swap rate’
- Lend ⇔ ‘Put on forward swap rate’

- Model forward swap rate at expiry as lognormal
- ⇒ Black-Scholes type formula





Black-Scholes for swaptions

$$V(0) = \text{PVBP}(0)\left\{S(0)\Phi(d_1) - K\Phi(d_2)\right\}$$

$$d_{1,2} = \frac{\ln\left(S(0)/K\right) \pm \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

PVBP(0) : value of annuity

$S(0)$: time - zero swap rate

Φ : cumulative normal distribution function

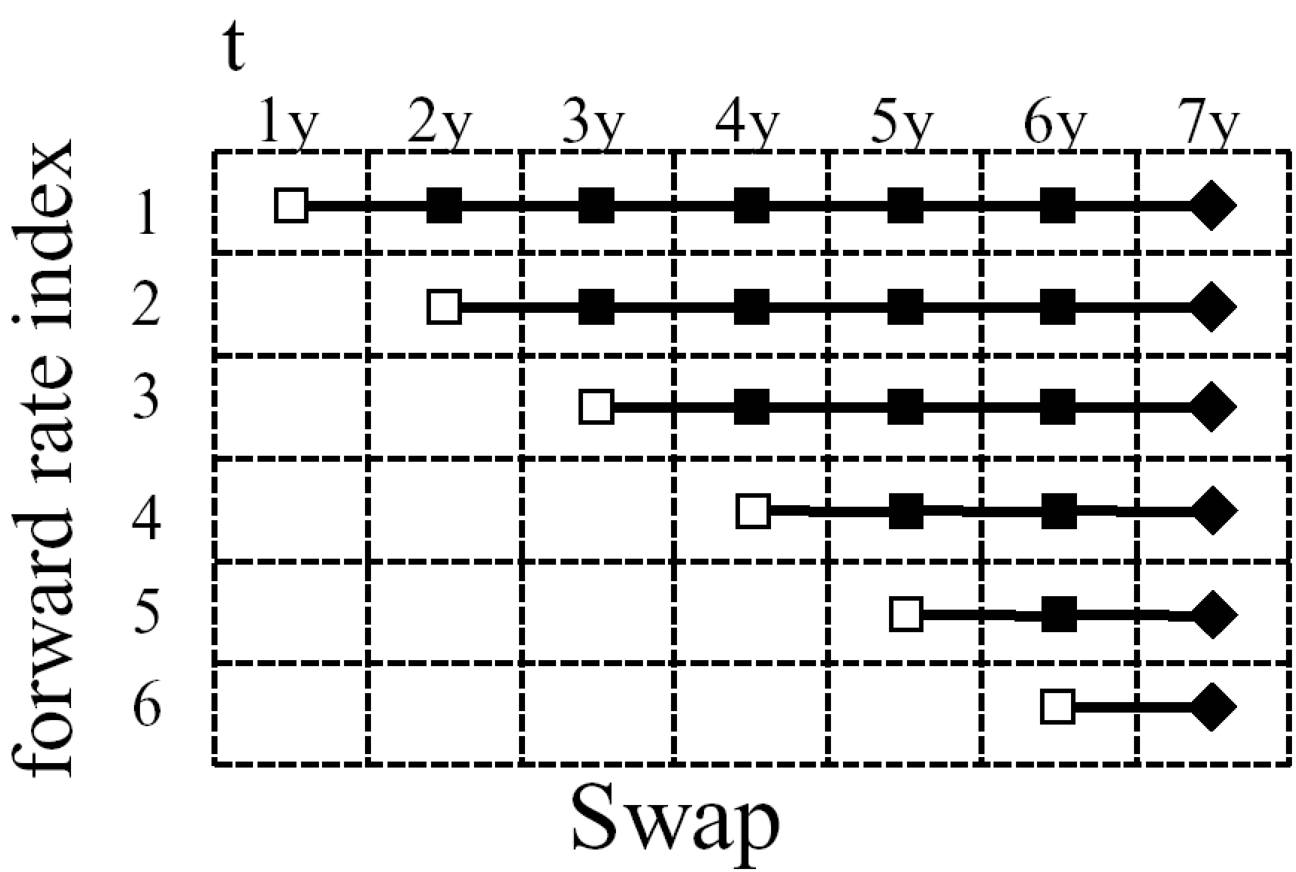
K : strike rate

T : expiry

σ : volatility of log swap rate

Bermudan swaptions

- Bermudan option:
- Exercisable at discrete set of time points



Model practice:
Model as interpolation tool

- Need model to price & hedge over-the-counter (OTC) Bermudan swaptions
- Features multiple swap rates
- Calibrated to relevant European swaption prices
- $\Rightarrow$ Volatility of forward swap rate = quoted

- Some models do not model stochastic volatility
- TS of IR $\Rightarrow$ state variable of model
- Re-calibrate model each day to volatilities

Hedging (bucket hedging)

- Offset risk by taking opposite position in underlying product
- For delta: Price sensitivity wrt swap rate
- Use underlying swaps to hedge
- $\Rightarrow$ justified theoretically

- Vega: sensitivity of price wrt calibration input volatility
- Financial engineering trick

$$V - \left(\frac{\partial V}{\partial \sigma_i} \Big/ \frac{\partial O_i}{\partial \sigma_i} \right) O_i$$

One-factor vs multi-factor models

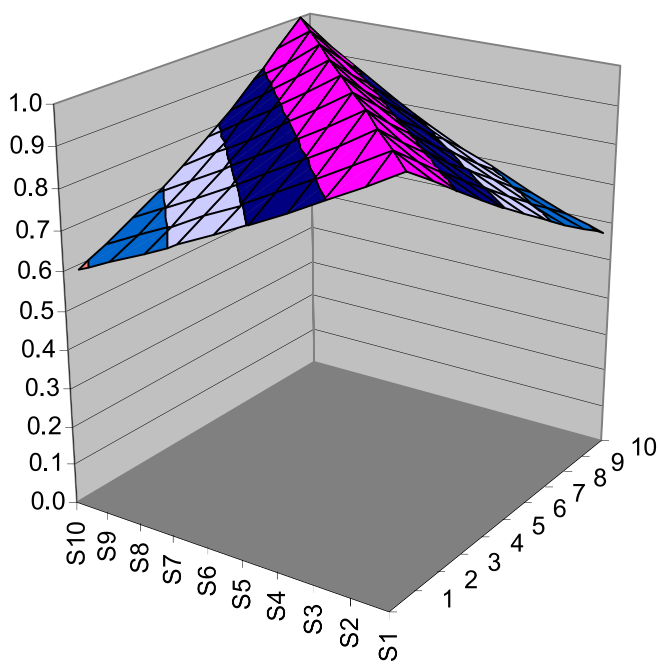
- Short-rate models
- Markov-functional (MF) models
- LIBOR & swap market models

- Accurate full correlation modeling possible in market models, not in short-rate/MF
- More accurate correlation modeling $\Rightarrow$ significantly better hedge results?
- Possible in MF to model relevant parts of correlation

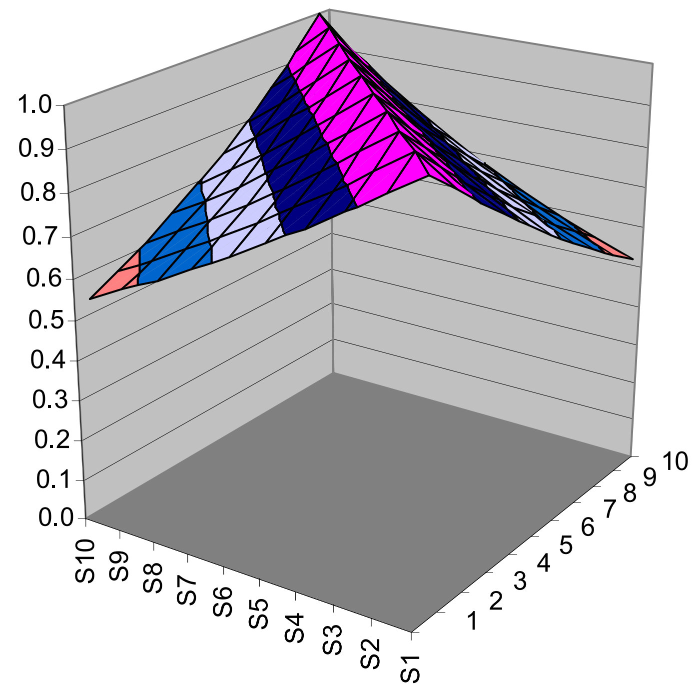
- Short-rate/MF more tractable

Impact correlation on pricing

Bid correlation



Ask correlation



Results from literature

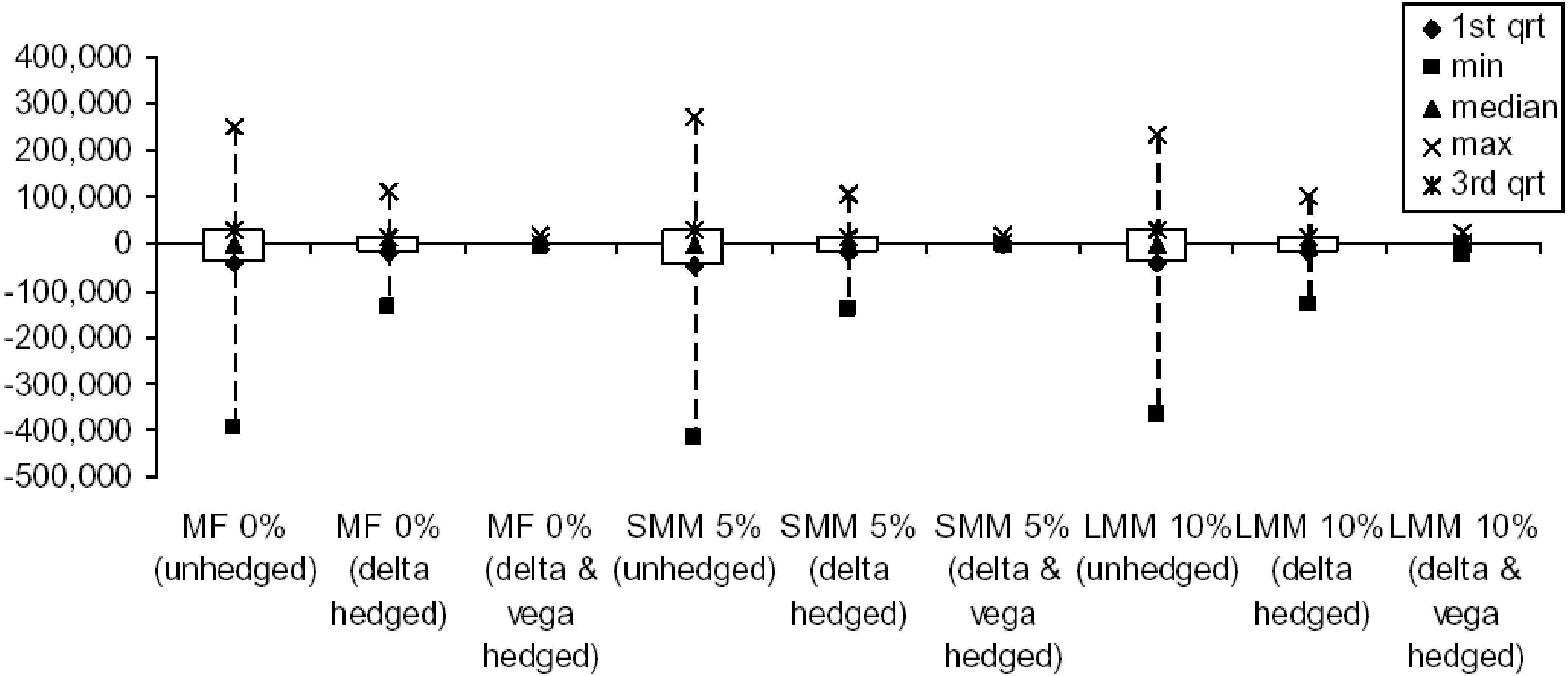
- Driessen, Klaassen & Melenberg (JFQA, 2003)
- Delta hedging for European options

- We look at both delta and delta&vega hedging
- 10Y Bermudan swaption deal
- USD data, 16-Jun-2003 – 16-Jun-2004
- Swap rates & ATM swaption volatility

- Markov-functional
- LIBOR market model
- Swap market model

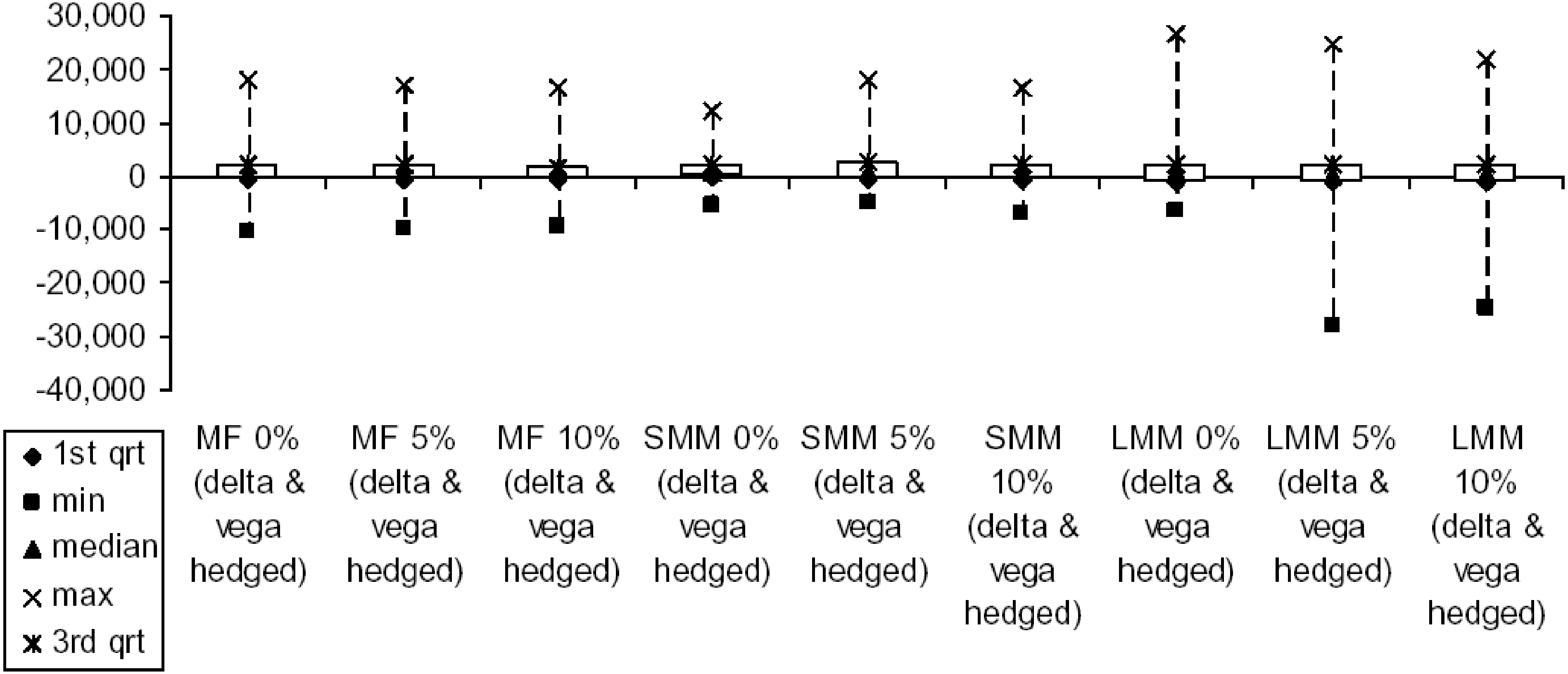
Expected result: Delta vs delta&vega hedging

- Delta&vega hedging outperforms delta hedging



Controversial result: Number of factors & correlation

- Number of factors & correlation do not seem to have significant impact on hedging



Part 2:

Rank reduction of correlation matrices

Based on:
Joint work with Igor Grubišić
&
Joint work with Patrick Groenen

Outline

- Calibration problem
- Mathematical formulation
- Majorization
- Geometric programming
- Efficiency comparison results

Calibration problem: LIBOR market model

- # stochastic factors needed for model to fit to correlation matrix is equal to rank
- Rank can be as high as dimension
- Not uncommon dimension = 80 or higher

- Reasons for using less factors:

1. Real-world # (significant) factors certainly not that high
2. Simply draw less random numbers

Mathematical formulation

Given:

1. Symmetric $n \times n$ matrix C, unit diagonal

2. Desired rank d

Find: Symmetric $n \times n$ matrix X

To minimize: $\|C - X\|^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij} \left(C_{ij} - X_{ij}\right)^2$

Such that: $\text{rank}(X) \le d$, $X \succeq 0$, X unit diagonal

Methods in the Literature: 17 publications and counting …

- Brigo, D. (2002), A note on correlation and rank reduction, Downloadable from www.damianobrigo.it.
- Brigo, D. & Mercurio, F. (2001), Interest Rate Models: Theory and Practice, Springer, New York
- Flury, B. (1988), Common Principal Components and Related Multivariate Models, J. Wiley & Sons, New York.
- Grubišić, I. & Pietersz, R. (2004), Efficient rank reduction of correlation matrices, Working paper, Utrecht University, Utrecht, Downloadable from www.few.eur.nl/few/people/pietersz.
- Higham, N. J. (2002), 'Computing the nearest correlation matrix–a problem from finance', IMA Journal of Numerical Analysis 22, 329–343.
- Hull, J. C. & White, A. (2000), 'Forward rate volatilities, swap rate volatilities, and implementation of the LIBOR market model', Journal of Fixed Income 3, 46–62.
- Morini, M. & Webber, N. (2004), 'An EZI method to reduce the rank of a correlation matrix'.
- Pietersz, R. & Groenen, P. J. F. (2005), 'Rank reduction of correlation matrices by majorization,' Forthcoming in Quantitative Finance.
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- Rapisarda, F., Mercurio, F. & Brigo, D. (2002), Parametrizing correlations: A geometric interpretation, Banca IMI Working Paper, Downloadable from www.fabiomercurio.it.
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- Rebonato, R. (1999b), Volatility and Correlation in the Pricing of Equity, FX and Interest-Rate Options, J. Wiley & Sons, Chichester.
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- Sidenius, J. (2000), 'LIBOR market models in practice,' Journal of Computational Finance 3(3), 5–26.
- Weigel, P. (2004), 'Optimal calibration of LIBOR market models to correlations,' Journal of Derivatives 12(3).
- Wu, L. (2003), 'Fast at-the-money calibration of the LIBOR market model using Lagrange multipliers', Journal of Computational Finance 6(2), 39–77.
- Zhang, Z. & Wu, L. (2003), 'Optimal low-rank approximation to a correlation matrix', Linear Algebra and its Applications 364, 161–187.

Majorization algorithm: Straightforward to implement

- for k = 0, 1, 2 do
  - stop if convergence criterion satisfied
  - for i = 1, 2, …, n do
    - Set B := $\sum_{j \neq i} w_{ij} \mathbf{x}_j \mathbf{x}_j^T$
    - Set λ := largest eigenvalue of dxd matrix B
    - Set z := $\lambda \mathbf{x}_i - \mathbf{B}\mathbf{x}_i + \sum_{j \neq i} w_{ij} r_{ij} \mathbf{x}_j$
    - If z $\neq$ 0, set i-th row of X equal to z/||z||
  - end for
- end for

- Global convergence guaranteed

Geometric programming: Correlation matrix as inner product matrix of a configuration

- Let Y be nxd matrix, such that $||Y_i||=1$
- Then nxn matrix X,
- $X_{ik} = \langle Y_i, Y_k \rangle$
- Is a correlation matrix of rank d
- If X is correlation matrix of rank d, then associated Y can be found
- Freedom of orthogonal transformation

Geometric programming

- Manifold: $S^0 \times S^1 \times \ldots \times S^{d-1} \times \underbrace{S^{d-1} \times \ldots \times S^{d-1}}_{(n-d)\times}$

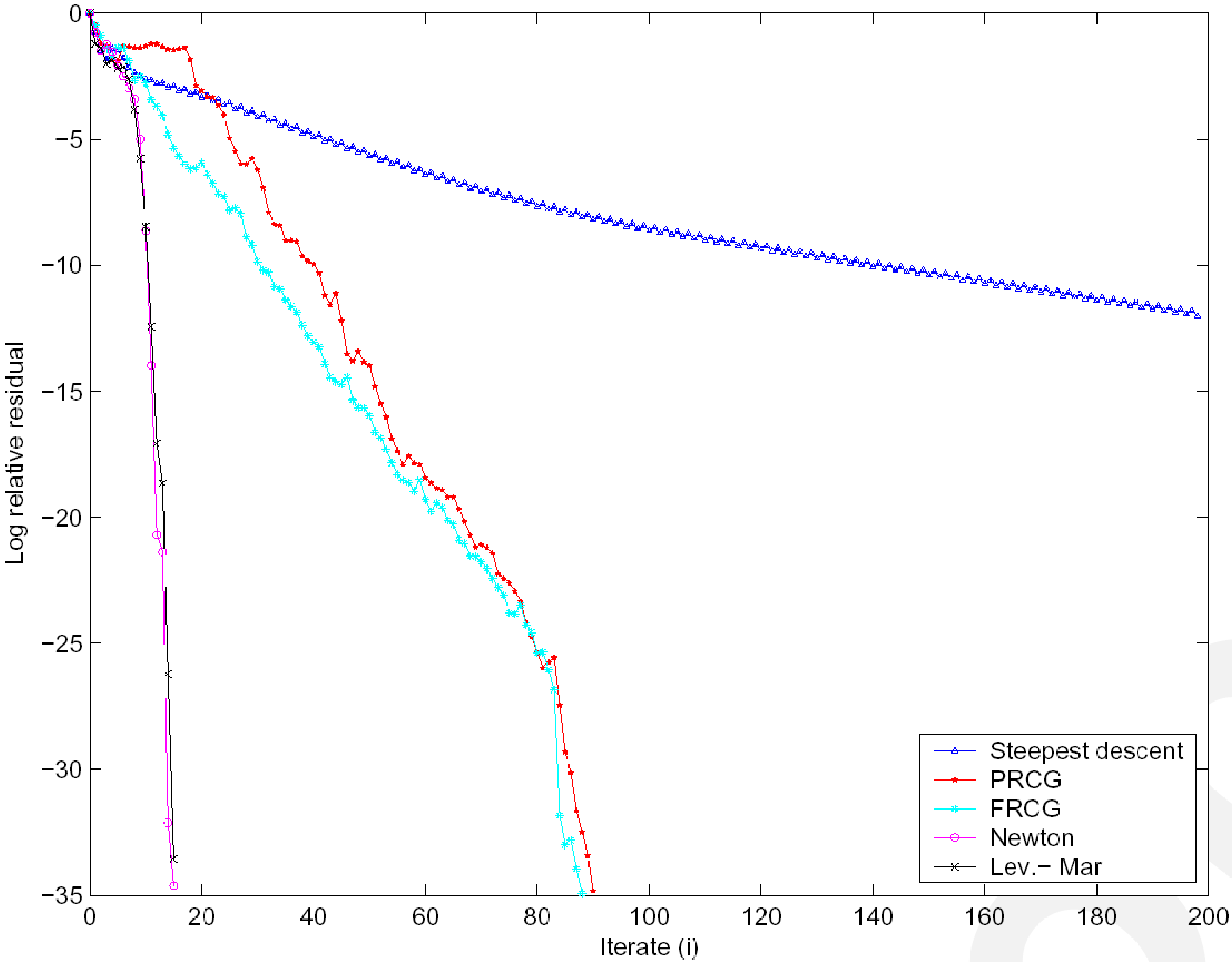
$$Y = \underbrace{\left.\begin{pmatrix} \times & 0 \cdots & 0 \\ & & \vdots \\ & \times & 0 \\ & & \times \\ & \otimes & \\ & & \end{pmatrix}\right\} n}_{d}$$

- Optimization over curved space (manifolds)
- Formulate Hessian, gradient, ‘straight lines’ etc in terms of differential geometric means
- $\Rightarrow$ Apply Newton or conjugate gradient

- Benefit: Hessian & gradient assume their ‘natural’ forms
- $\Rightarrow$ more efficient to calculate

- MATLAB implementation ‘LRCM MIN’ available:
- www.few.eur.nl/few/people/pietersz/

Fast convergence



MATLAB demonstration

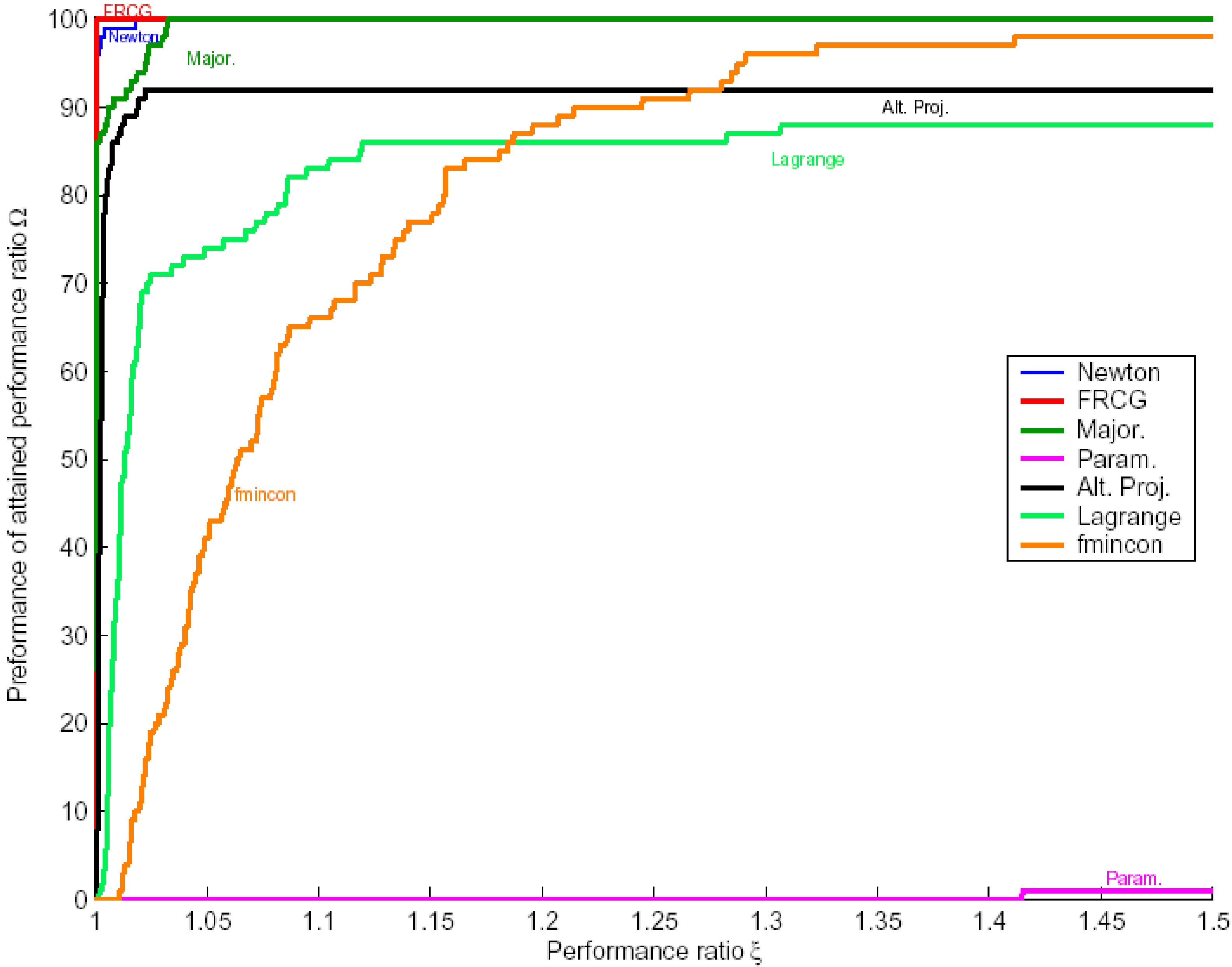
- **3x3 correlation matrices**

$$C = \begin{pmatrix} 1 & x & y \\ x & 1 & z \\ y & z & 1 \end{pmatrix}$$

- **Turns out: rank(C) ≤ 2 & C is p.s.d. condition is equivalent to**

$$0 = \det(C) = -\left\{ x^2 + y^2 + z^2 \right\} + 2xyz + 1$$

Performance profile, n=60, d=5, t=3



Conclusions

- Part I:
- Interesting hedge tests that have far-reaching implications for use of models in practice

- Part II:
- Majorization: Quite efficient, easy to implement
- Geometric programming: Efficiency champion

- Papers downloadable from:
- www.few.eur.nl/few/people/pietersz/