Importance Sampling for Credit Risk Economic Capital

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Agenda

- Economic Capital
  - Concept and purpose
  - RAROC and risk/reward decision making
- Economic Capital for Credit Risk
- Importance sampling
  - Application to Credit Risk EC
- Numerical illustration
- Conclusions and further research

Economic Capital

Banking risks

- A bank exposes itself to several risks with the aim of generating attractive returns:
  - credit risk
  - market risk
  - interest-rate risk

- As a consequence, the bank is also exposed to other risks:
  - operational risk
  - business risk

Banking risks

- Questions:
  - Is the risk taken (potentially leading to losses for the bank) commensurate with the available capital?
  - How to choose between an exposure to - say - market and credit risk?

- Need for a risk measure that is comparable across risk types and business units.

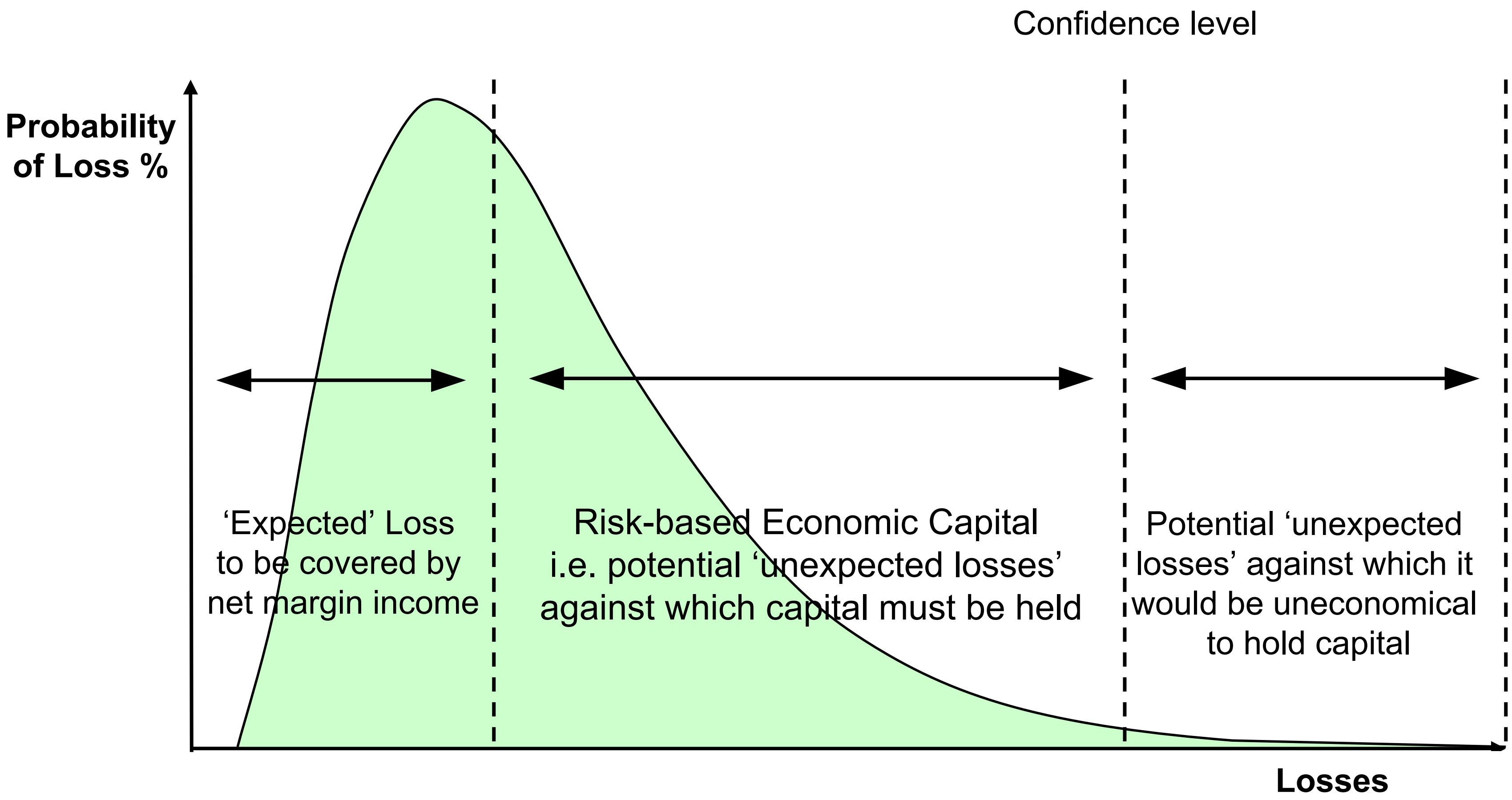
- Economic Capital is such a risk measure

Economic Capital

- Economic capital:
  - the amount of capital the Bank should possess to be able to sustain larger than expected losses with a high degree of certainty, given the risk exposures of the Bank.

- Choices:
  - Time horizon: typically one year
  - Degree of certainty (“confidence level”): related to the desired risk profile of the Bank, as reflected in its credit rating.

Economic Capital

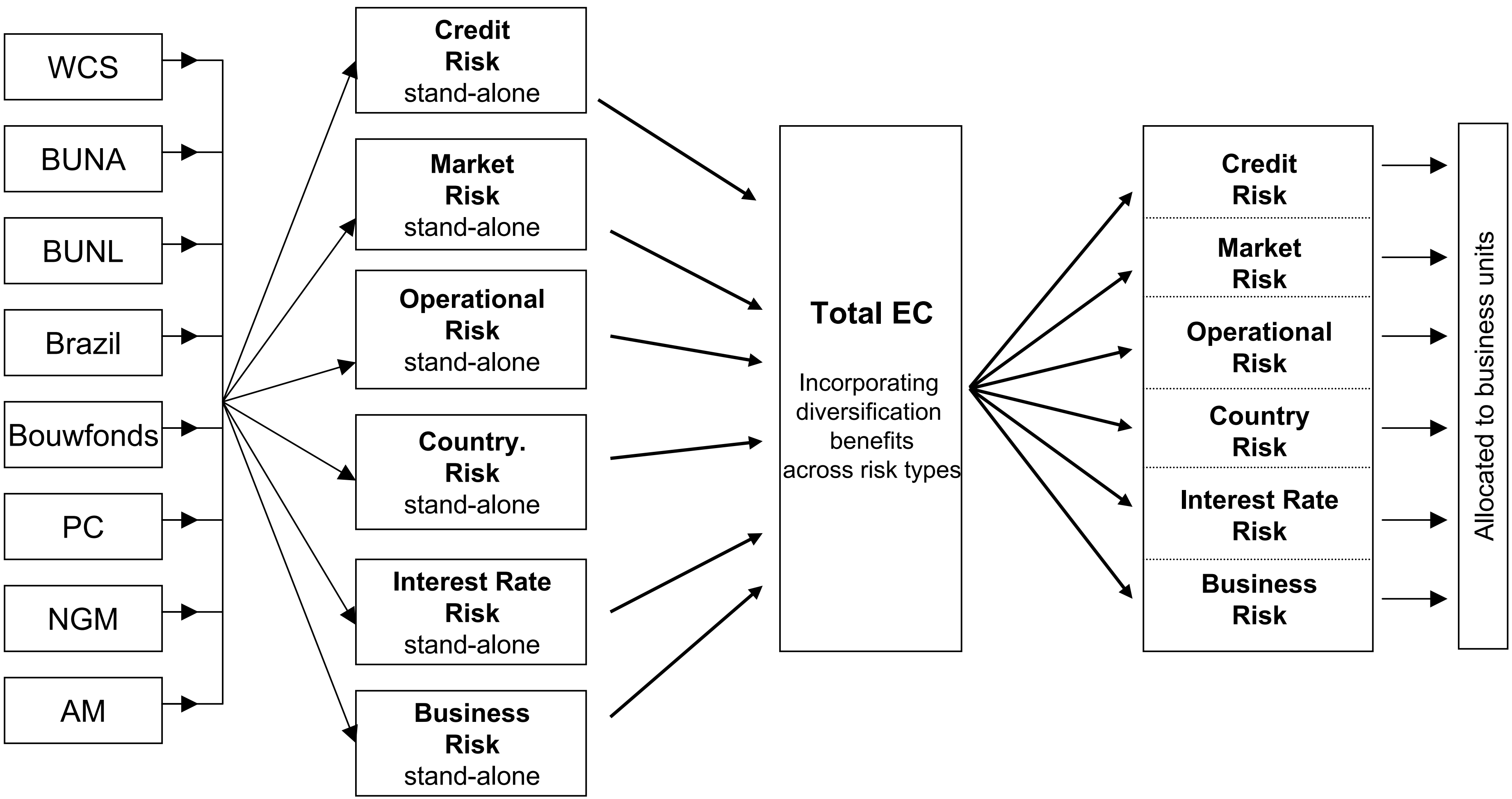


Economic Capital

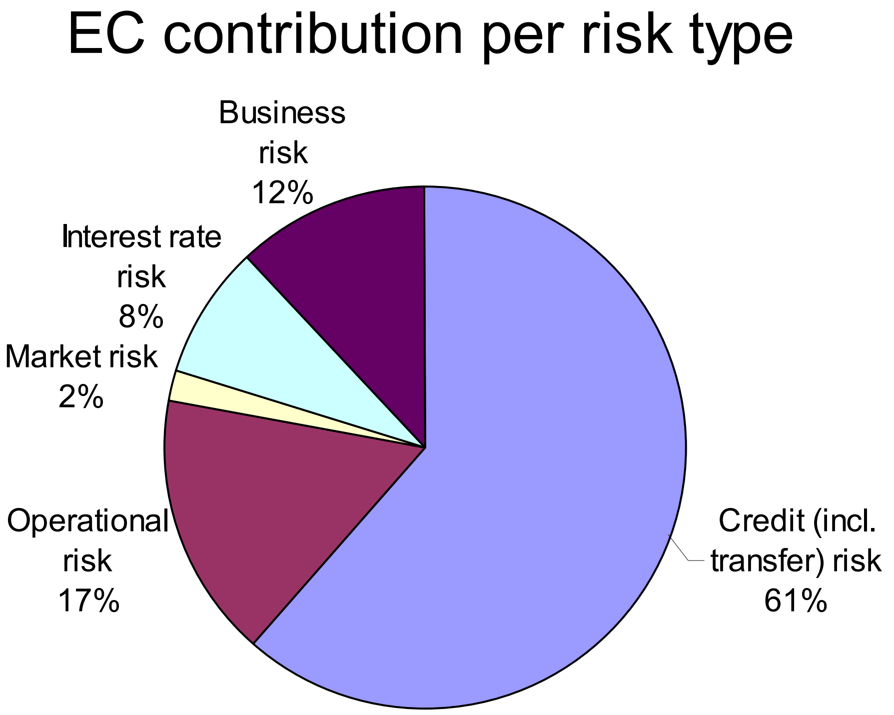
- Loss distribution should take into account diversification:
  - across business units
  - across risk types

- Practical approach:
  - Determine EC for each risk type separately, including diversification effects between business units
  - Aggregate EC per risk type over risk types, including diversification effects between risk types.

Calculation of Economic Capital



Economic Capital ABN AMRO



Risk-reward decision making

- Risk-Adjusted Return On Capital:

$$\text{RAROC} = \frac{\text{Revenues} - \text{Cost} - \text{Expected Loss}}{\text{Economic Capital}}$$

- Business activities evaluated and compared on the basis of RAROC:
  - EC in RAROC formula represents contribution to total EC

Credit Risk Economic Capital

Credit Risk

- Credit risk arises because clients (borrowers) may default on their obligations
- Potential losses are function of:
  - default probability of borrower
  - size of obligation at default
  - recovery in case of default
  - correlation between likelihood of default of different borrowers
- Company's credit quality usually assessed with credit rating (external or internal)
- Better rating corresponds to lower probability of default

Credit Risk Economic Capital

- L_i denotes loss on loan i, $i = 1, \dots, N$
- L_i are correlated
- Portfolio loss $L = \sum_{i=1}^{N} L_i$
- Economic capital $EC = L^{(0.9995)} - E[L]$
- Economic capital contribution of loan i

$$ECC_i = E[L_i \mid L = L^{(0.9995)}] - E[L_i]$$

- Note: $\sum_{i=1}^{N} ECC_i = EC$

Modelling correlated losses

- Correlated defaults:
  - Associate normal random variable $S_i \sim N(0,1)$ with loan i
  - Default occurs if realisation of S_i falls below threshold

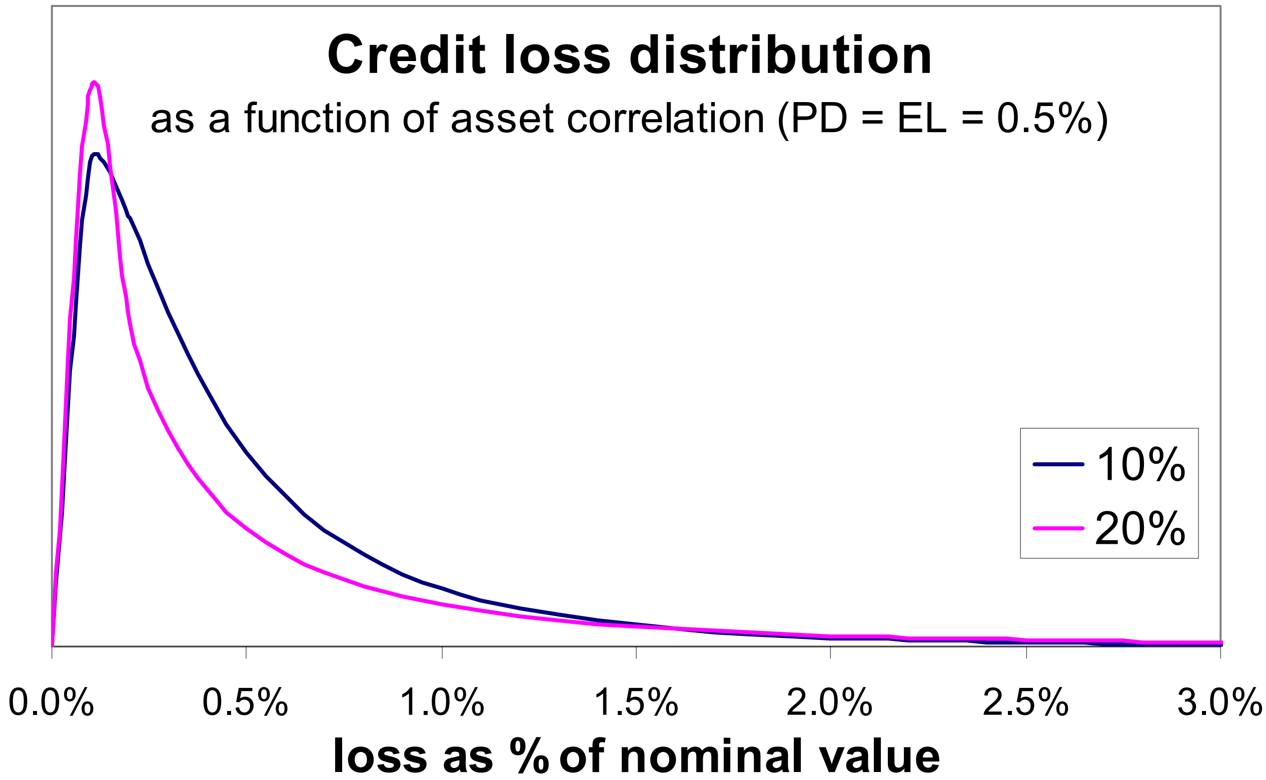
    $$S_i \leq N^{-1}(PD)$$

    This leads to loss L_i
  - Correlate random variables S_i of different clients. This induces correlation between losses.
  - Correlation usually modelled through factor model. Simplest form is one-factor model:

    $$S_i = \sqrt{\alpha_i}\, F + \sqrt{(1 - \alpha_i)}\, Y_i$$

    with $Y_i \sim N(0,1)$ independent of $F \sim N(0,1)$.

Illustration: Effect of correlation



Loss quantile $L^{(0.9995)}$ = 5.3% if α_i = 10%
10.8% if α_i = 20%

Importance Sampling

Based on work with Erwin Charlier, Koen Veltman, and Jasper van den Eshof

Standard Monte Carlo

- Portfolio loss in scenario s is $L^s = \sum_{i=1}^{N} L_i^s$
- Loss distribution F_L is approximated as

$$F_{L,M}(x) = 1 - \frac{1}{M}\sum_{s=1}^{M} I_{\{L^s > x\}}$$

- $L^{(p)}$ can be estimated by

$$L^{(p)}_{F_{L,M}} = \underset{x}{\operatorname{argmin}}\left\{F_{L,M}(x) \geq p\right\}$$

- For large p (close to 1), many scenarios are needed for a reasonably precise estimate of $L^{(p)}$

Importance Sampling

- Idea is to modify the sampling distribution in such a way that we sample more portfolio losses in the tail of the loss distribution.
- Let λ^s denote the likelihood ratio of the true sampling measure and the importance sampling measure in scenario s. Then F_L can be approximated as

$$F_{L,M,\lambda}(x) = 1 - \frac{1}{M} \sum_{s=1}^{M} I_{\{L^s > x\}} \lambda^s$$

and $L^{(p)}$ as

$$L^{(p)}_{F_{L,M,\lambda}} = \underset{x}{\arg\min} \{ F_{L,M,\lambda}(x) \geq p \}$$

Importance sampling (2)

- One can prove (Glynn [1996]) that:
  - $L^{(p)}_{F_{L,M,\lambda}}$ converges to $L^{(p)}$ if $M \to \infty$
  - the variance of $L^{(p)}_{F_{L,M,\lambda}}$ is small if we can find a precise estimator of the tail probability 1-*p*

$$1 - p = E[I_{\{L > L^{(p)}\}}] = E_{G_L}[I_{\{L > L^{(p)}\}} \lambda]$$

with G_L representing the credit loss distribution that results from sampling from the importance sampling distribution.

Choice of IS distribution

- Shift mean of risk factor distributions to $\mu < 0$.
- Choice of μ ideally so as to minimize variance of importance sampling estimator of tail probability:
  - feasible for one-factor model (Kalkbrener *et al.* [2004])
  - not possible for multi-factor model
- 2-step Monte Carlo:
  - standard Monte Carlo to obtain estimate $L^{(p)}_{F_{L,M}}$ of $L^{(p)}$
  - set μ equal to average of risk factor realisations in some subset of scenarios that yield losses close to $L^{(p)}_{F_{L,M}}$

    $$\overline{S} = \left\{ s \mid L^{s} \in [0.9 \cdot L^{(p)}_{F_{L,M}}, 1.1 \cdot L^{(p)}_{F_{L,M}}] \right\}$$
  - importance sampling by using shifted risk-factor distributions

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EC Contributions

- Economic capital contribution of loan or sub-portfolio *i*

$$ECC_i = E[L_i \mid L = L^{(0.9995)}] - E[L_i]$$

- Estimate of ECC_i using standard MC simulation:

$$\widehat{ECC}_i = \frac{1}{\left|\bar{S}\right|} \sum_{s \in \bar{S}} L_i^s - E\left[L_i\right]$$

- Estimate of ECC_i using importance sampling:

$$\widehat{ECC}_i = \frac{\sum_{s \in \bar{S}} L_i^s \lambda^s}{\sum_{s \in \bar{S}} \lambda^s} - E\left[L_i\right]$$

Illustration

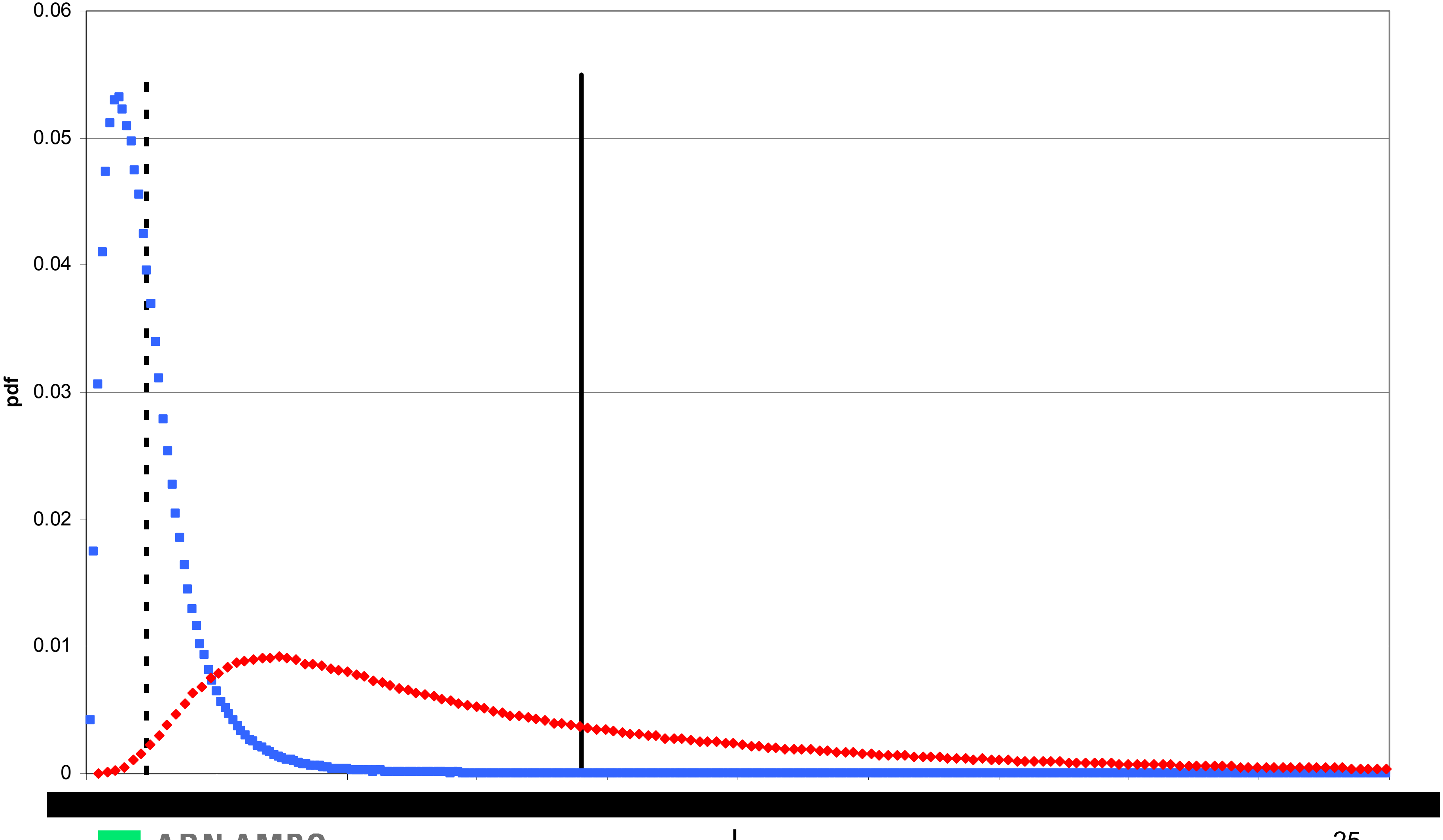
Settings

- Portfolio of 25,000 loans
- Common risk factors represent combinations of
  - countries / regions (12)
  - industry sectors (50)
- 1 million Monte Carlo scenarios *(M)*
- Interval used around EC estimate to determine shifted mean of risk factors, as well as for EC contributions

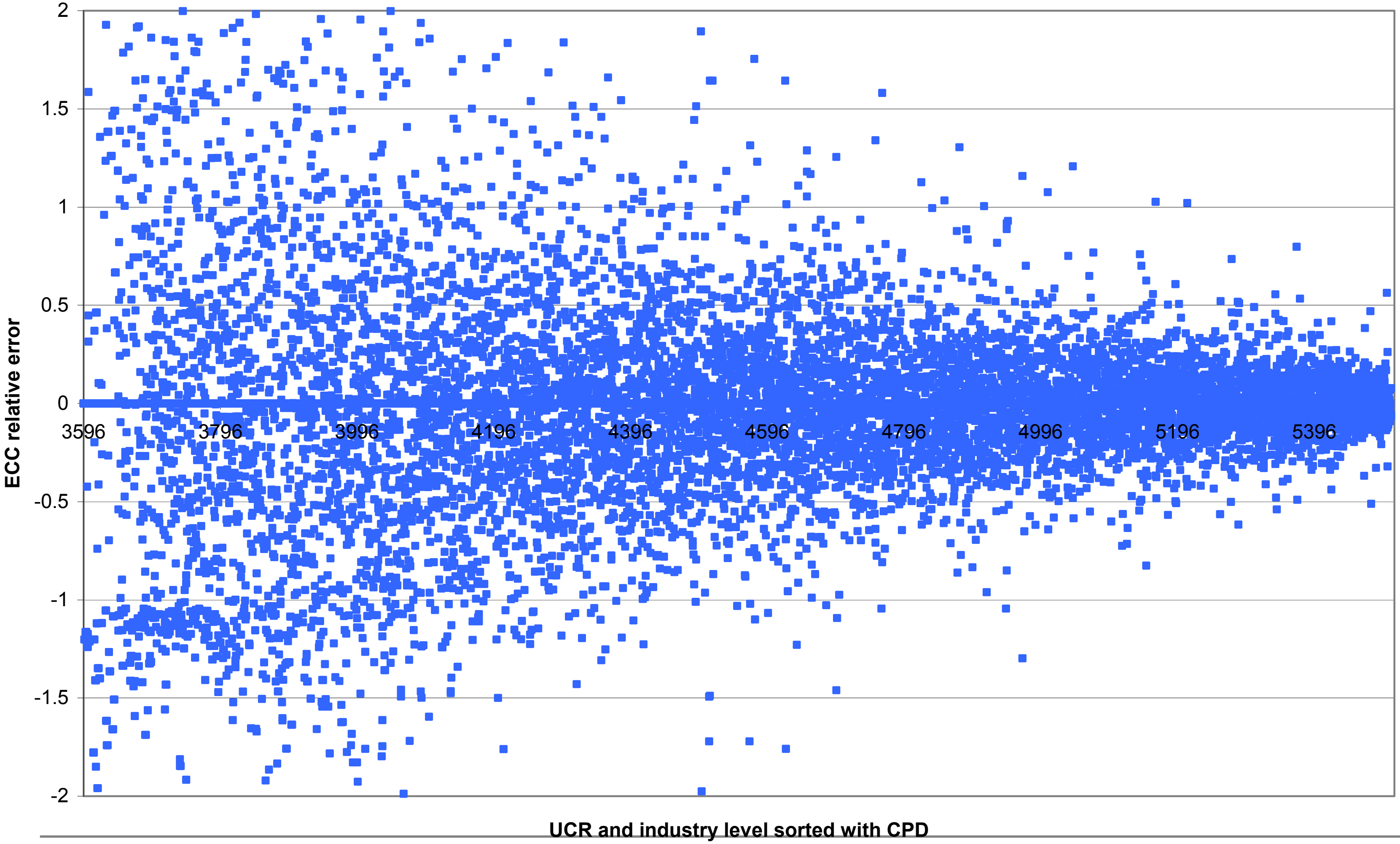
$$\overline{S} = \left\{ s \mid L^s \in [0.9 \cdot L^{(p)}_{F_{L,M}}, 1.1 \cdot L^{(p)}_{F_{L,M}}] \right\}$$

- Variance reduction for estimate of
  - EC: order of 100
  - ECC per region & industry combination: order of 10

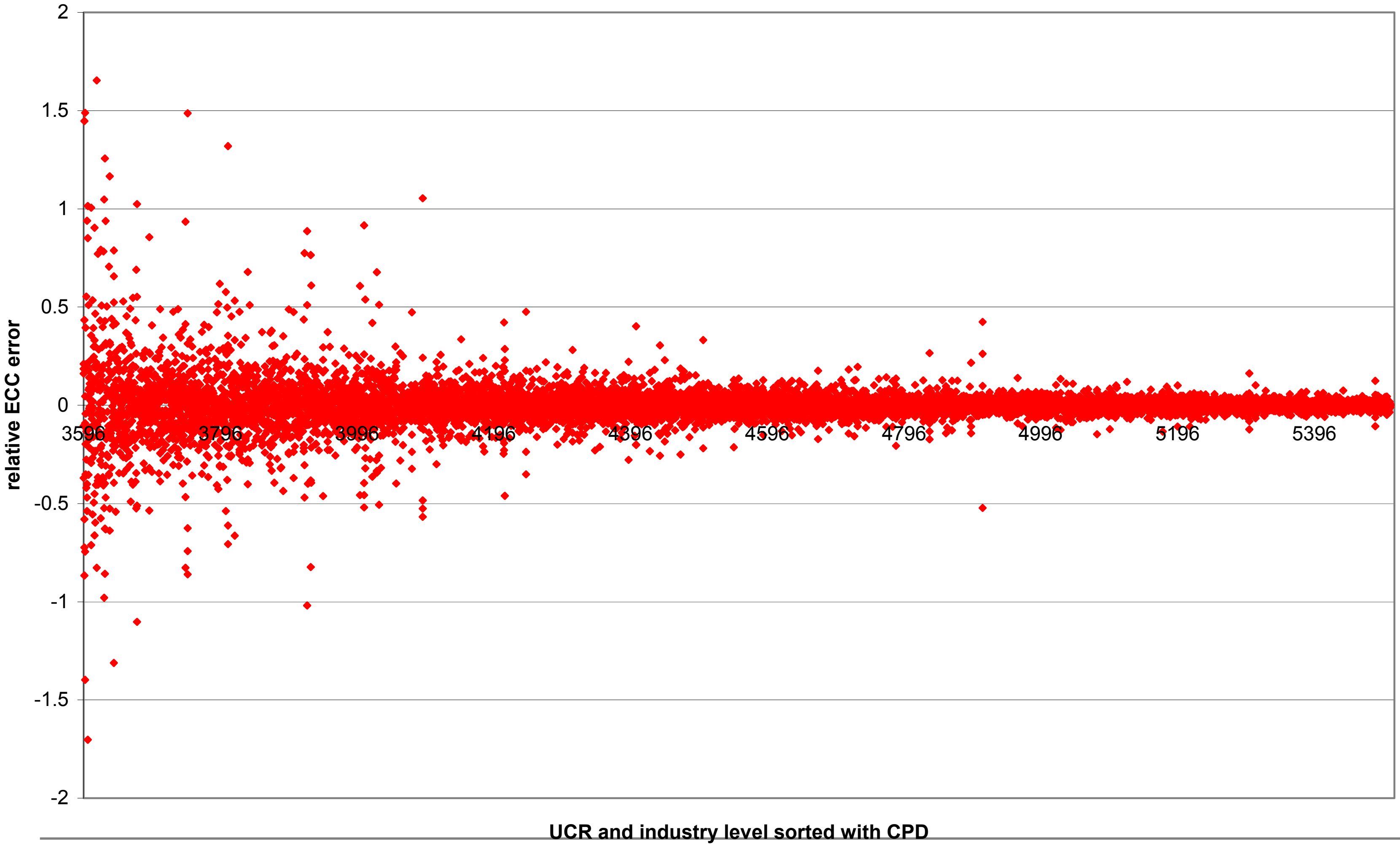
Loss distributions



ECC with Standard Monte Carlo



ECC with 2-step Monte Carlo



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Conclusions and further research

Conclusions

- Focus on both EC and ECC
- High loss quantile: standard MC not suitable
- Importance sampling: shift mean of risk factors
  - 2-step Monte Carlo
- EC: substantial improvement
- ECC: improvement but need more in practice

Further research

- Determination of optimal shift:
  - Use of one-factor approximation (Kalkbrener *et al* [2004])
  - Refinements to 2-step Monte Carlo (choice of interval, sensitivity of optimal shift to portfolio composition)
- Alternative IS distributions:
  - Increase in variance instead of shift in mean (Morokoff [2005])
- IT: parallel computing