# Beating the curse of dimensionality in options pricing and optimal stopping

# (joint work with Ph.D. student Yilun Chen)

Cornell

LNMB

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Punchline	$\begin{array}{c} \textbf{Model} + \textbf{Problem} \\ \text{ooo} \end{array}$	Intuition + Main results	Future Research
Outline			









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2 Model + Problem

Intuition + Main results



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Punchline ○●○	Model + Problem	Intuition + Main results	Future Research
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#### • Opt. pricing and opt. stopping central to Fin. Math, OR, AP

- Rich history in NL  $\rightarrow$  Amsterdam Bourse in 1600's
- Many real-world comp. problems have ....
  - High-dim process generating.7
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- Seems theoretically intractable, vast lit.
- Opt. dual martingales hard to understand explicitly
   Many approaches need fully pested cond. expl

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Backwards induction

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#### • We prove ...

- $\exists$  "simple and elegant formula" for opt. stop. as  $\infty$  sum
- Even better, if you truncate the sum after k terms ...
  - Error bounded by a Easy to simulate!
- Yields efficient rand. ε-optimal algorithms
  - Continue Trev(-)
  - Data-driven : 7<sup>poly(1)</sup> samples
  - Even with high-dim and path-dependence
  - Beats the curse of dimensionality!
  - (sample-based) PTAS for gen. opt. stop
- New connection to net. flows
  - Yields simple explicit dual mart sol.
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- We prove ...
  - $\bullet~\exists$  "simple and elegant formula" for opt. stop. as  $\infty$  sum
  - Even better, if you truncate the sum after k terms ...
    - Error bounded by  $\frac{1}{k}$
    - Easy to simulate!
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Intuition + Main results



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Model			

#### Discrete time : T periods

- $\mathcal{F} = \{\mathcal{F}_t, t \in [1, T]\}$  gen. by *D*-dim process  $\mathbf{Y} = Y_1, \dots, Y_T$ • Cost functions  $\{g_t, t \in [1, T]\}$ 
  - \*  $Z_t = g_t(Y_{(\eta)}) = \cos t$  of stop. at t \* Non-neg. + integrable
- T: set of F-adapted stop.times in [1, T]
- Note: disc. time, won't dwell on pathologies
  - Assume all conditionings etc. well-defined.
- Note: Sometimes assume  $Z_t \in [0, 1]$  or [0, U]
  - Simplifies notations etc.
  - Could state in terms of u.b., trunc., etc.

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  - $= \{g_t, t \in [1, T]\}$
  - Non-neg. Integrable
- $\mathcal{T}$ : set of  $\mathcal{F}$ -adapted stop.times in [1,  $\mathcal{T}$ ]
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- Cost functions  $\{g_t, t \in [1, T]\}$ 
  - $Z_t \triangleq g_t(Y_{[t]}) = \text{cost of stop. at t}$
  - Non-neg. + integrable
- T: set of F-adapted stop.times in [1, T]
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Punchline	Model + Problem ○●○	Intuition + Main results	Future Research
Model			

- Discrete time : T periods
- $\mathcal{F} = \{\mathcal{F}_t, t \in [1, T]\}$  gen. by *D*-dim process  $\mathbf{Y} = Y_1, \dots, Y_T$
- Cost functions  $\{g_t, t \in [1, T]\}$ 
  - $Z_t \stackrel{\Delta}{=} g_t(Y_{[t]}) = \text{cost of stop. at t}$
  - Non-neg. + integrable
- $\mathcal{T}$ : set of  $\mathcal{F}$ -adapted stop.times in [1, T]
- Note: disc. time, won't dwell on pathologies
  - Assume all conditionings etc. well-defined.
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 $Model + Problem \\ \circ \circ \bullet$ 

Intuition + Main results

Future Research

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#### **Problem statement**

- Full path-dependence
- High dimensionality
- Massive state space
- Gen. disc-time optimal stopping
- Pricing Bermudan Options
- Fundamental problem in control theory
- WLOG generally discuss the min. problem
  - Can transform max to min
  - Ideas are clearer for min
- Once you have gen. opt. stop. you have a lot more
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Intuition + Main results ○●○○○○○○○ Future Research

# Main intuition

# • OPT $\stackrel{\Delta}{=} \inf_{\tau \in \mathcal{T}} \boldsymbol{E}[\boldsymbol{Z}_{\tau}]$

- OPT =  $\inf_{\tau \in \mathcal{T}} E[Z_{\tau}] \ge E[\min_{t \in [1, \mathcal{T}]} Z_t]$
- OPT =  $E\left[\min_{t \in [1,T]} Z_t\right] + \inf_{\tau \in \mathcal{T}} E\left[Z_{\tau} E\left[\min_{i \in [1,T]} Z_i | \mathcal{F}_{\tau}\right]\right]$
- $Z_t^1 \stackrel{\Delta}{=} Z_t$  ,  $Z_t^2 \stackrel{\Delta}{=} Z_t^1 E[\min_{i \in [1,T]} Z_i^1 | \mathcal{F}_t]$
- OPT =  $\inf_{\tau \in \mathcal{T}} E[Z_{\tau}^1] = E[\min_{t \in [1,T]} Z_t^1] + \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^2]$
- $Z_t^3 \stackrel{\Delta}{=} Z_t^2 E[\min_{i \in [1,T]} Z_i^2 | \mathcal{F}_t]$
- OPT =  $E\left[\min_{t \in [1,T]} Z_t^1\right] + E\left[\min_{t \in [1,T]} Z_t^2\right] + \inf_{\tau \in T} E[Z_{\tau}^3]$
- $Z_t^{k+1} = Z_t^k E[\min_{i \in [1,T]} Z_i^k | \mathcal{F}_t]$
- OPT =  $\sum_{k=1}^{\infty} E[\min_{i \in [1,T]} Z_i^k] + \lim_{k \to \infty} \inf_{\tau \in T} E[Z_{\tau}^k]$
- THEOREM : OPT =  $\sum_{k=1}^{\infty} E[\min_{t \in [1,T]} Z_t^k]!$

 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \circ \circ \circ \end{array}$ 

Intuition + Main results ○●○○○○○○○ Future Research

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Intuition + Main results ○●○○○○○○○ Future Research

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Intuition + Main results ○●○○○○○○○ Future Research

#### Main intuition

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Intuition + Main results ○●○○○○○○○ Future Research

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• OPT  $= E[\min_{t \in [1,T]} Z_{t}^{1}] + E[\min_{t \in [1,T]} Z_{t}^{2}] + \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^{3}]$   
•  $Z_{t}^{k+1} = Z_{t}^{k} - E[\min_{i \in [1,T]} Z_{i}^{k} | \mathcal{F}_{t}]$   
• OPT  $= \sum_{k=1}^{\infty} E[\min_{i \in [1,T]} Z_{i}^{k}] + \lim_{k \to \infty} \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^{k}]$   
• THEOREM : OPT  $= \sum_{k=1}^{\infty} E[\min_{t \in [1,T]} Z_{t}^{k}]!$ 

 $\underset{000}{\text{Model}} + \text{Problem}$ 

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• OPT 
$$\triangleq \inf_{\tau \in \mathcal{T}} E[Z_{\tau}]$$
  
• OPT  $= \inf_{\tau \in \mathcal{T}} E[Z_{\tau}] \ge E[\min_{t \in [1,T]} Z_{t}]$   
• OPT  $= E[\min_{t \in [1,T]} Z_{t}] + \inf_{\tau \in \mathcal{T}} E[Z_{\tau} - E[\min_{i \in [1,T]} Z_{i}|\mathcal{F}_{\tau}]]$   
•  $Z_{t}^{1} \triangleq Z_{t}$ ,  $Z_{t}^{2} \triangleq Z_{t}^{1} - E[\min_{i \in [1,T]} Z_{i}^{1}|\mathcal{F}_{t}]$   
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 $\underset{000}{\text{Model}} + \text{Problem}$ 

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# A "formula" for opt. stopping : OPT =

$$E[\min_{i\in[1,T]}Z_i]+$$

$$E\left[\min_{i\in[1,T]} \left(Z_i - E\left[\min_{j\in[1,T]} Z_j | \mathcal{F}_i\right]\right)\right] + \\E\left[\min_{i\in[1,T]} \left(Z_i - E\left[\min_{j\in[1,T]} Z_j | \mathcal{F}_i\right] - E\left[\min_{j\in[1,T]} \left(Z_j - E\left[\min_{k\in[1,T]} Z_k | \mathcal{F}_j\right]\right) | \mathcal{F}_i\right]\right)\right] + \\$$

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# Fast convergence

# • THEOREM : $\left| \text{OPT} - \sum_{i=1}^{k} E[\min_{t \in [1,T]} Z_t^i] \right| \le \frac{U}{k+1}$

• Note : Also prove other bounds ind. of U (even if  $U = \infty$ )

#### Note : analysis tight in the worst-case

In many examples converges much faster.

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# Algorithmic implications

#### • $E[\min_{t \in [1,T]} Z_t^k]$ can be computed by sim!

- No curse of dimensionality!
- Completely data-driven
- But only need a few terms!
- Explicit runtime depends on assumptions + type of approx.
- In general  $\epsilon$ -approx for OPT in  $T^{\text{poly}(\frac{1}{\epsilon})}$  time! (w.h.p.)
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 $\underset{000}{\text{Model}} + \text{Problem}$ 

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# Algorithmic implications (cont.)

#### "Canonical" Theorem

- Suppose  $P(Z_t \in [0, 1]) = 1$  for all *t*.
- Then for all  $\epsilon, \delta \in (0, 1)$ ,  $\exists$  a rand. alg.  $A_{\epsilon, \delta}$  s.t. . . .

# $2^{O(\frac{1}{2})} \times 7^{O(\frac{1}{2})} \times \log(\frac{1}{2}),$

With only † calls to a simulator for Y (cond. on hist.),
 Returns r.v. X s.t.

#### $P(|X - OPT| \le \epsilon) \ge 1 - \delta.$

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In time

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 $\mathsf{P}(|X - \mathsf{OPT}| \le \epsilon) \ge 1 - \delta.$ 

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 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

Intuition + Main results

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# Algorithmic implications (cont.)

#### "Canonical" Theorem

- Suppose  $P(Z_t \in [0, 1]) = 1$  for all *t*.
- Then for all  $\epsilon, \delta \in (0, 1)$ ,  $\exists$  a rand. alg.  $A_{\epsilon, \delta}$  s.t. ...

In time

$$2^{O(rac{1}{\epsilon^2})} imes T^{O(rac{1}{\epsilon})} imes \log(rac{1}{\delta}),$$

• With only  $\uparrow$  calls to a simulator for **Y** (cond. on hist.),

• Returns r.v. X s.t.

$$P(|X - OPT| \le \epsilon) \ge 1 - \delta.$$

 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

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## The max-flow connection

#### A correspondence between opt. stop and min-cut

- Illustrate in trivial 3-stage problem
  - Driving process Y is 1-d , supp. {1,2}
  - Z<sub>t</sub> = payout if stop at time t = most recent Y
- $\bullet$  Complicated duality lit  $\rightarrow$  max-flow min-cut
- Expansion = iterative flow alg.
- Won't give all details of reduction
  - Will equate 2 stop: times with 2 cuts.
  - Idea is simple and intuitive
  - Previously overlooked, not focused on the "right" marts
  - $\sim$  Past marts yielded soln V subproblems  $\rightarrow$  comp. slow

 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

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 $\underset{000}{\text{Model}} + \text{Problem}$ 

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## The max-flow connection (cont.)

# τ : If Z<sub>1</sub> = 1, STOP ; else STOP at time 2 Cut value = 1P<sub>1</sub> + 1P<sub>21</sub> + 2P<sub>22</sub> = E[Z<sub>τ</sub>]

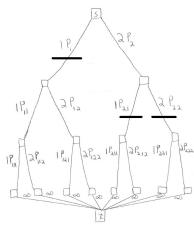
 $\underset{000}{\text{Model}} + \text{Problem}$ 

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## The max-flow connection (cont.)

•  $\tau$  : If  $Z_1 = 1$ , STOP ; else STOP at time 2 • Cut value =  $1P_1 + 1P_{21} + 2P_{22} = E[Z_{\tau}]$ 



 $\underset{000}{\text{Model}} + \text{Problem}$ 

Intuition + Main results

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## The max-flow connection (cont.)

•  $\tau$  : STOP when you see a 1 or the horizon ends

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 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \circ \circ \circ \end{array}$ 

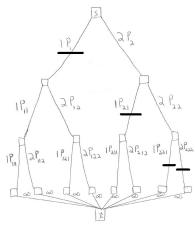
Intuition + Main results

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#### The max-flow connection (cont.)

•  $\tau$  : STOP when you see a 1 or the horizon ends

• Cut value =  $1P_1 + 1P_{21} + 1P_{221} + 2P_{222} = E[Z_{\tau}] = OPT$ 



 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

Intuition + Main results ○○○○○○○● Future Research

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# The max-flow connection (cont.)

#### • THEOREM : Solving opt.stop equal to solving min-cut

- OBS : Novel unification of many duality results for opt.stop
  - Karatzas, Kogan and Haugh, Rogers, Glasserman, ...
  - Opt. dual martingale  $\leftrightarrow$  max-flow
- OBS : Simple proofs and intuition about many past results
  - $\bullet~\mbox{Tree}~\mbox{network} \rightarrow \mbox{greedy works, block. flow is opt, } \ldots$
- OBS : Our algorithms can be interpreted as ...
  - Pastrand, iter method for max-flow on massive tree.
  - Amount pushed on a given edge in round k is .....
  - Total flow pushed in round k is  $E[\min_{t \in [1,1]} Z^t]$
- $\bullet$  OBS : Expansion  $\rightarrow$  simple and explicit opt. dual sol.
- $\bullet$  OBS : Expansion  $\rightarrow$  simple and explicit opt. stop. rule
  - Slop when 2(----2) [2], min<sub>el (0</sub>, 2/2].
     Slop when you reach sat, edge (min.ed).

 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

Intuition + Main results ○○○○○○○● Future Research

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  - $\bullet~\mbox{Opt.}$  dual martingale  $\leftrightarrow \mbox{max-flow}$
- OBS : Simple proofs and intuition about many past results
   Tree network -> greedy works, block, flow is ont
- OBS : Our algorithms can be interpreted as ...
  - Fast rand, iter, method for max-low on massive tree.
     Amount pushed on a given edge in round A is set.
  - Clotel flow pushed in round k is E[mintel J] Z
- $\bullet$  OBS : Expansion  $\rightarrow$  simple and explicit opt. dual sol.
- $\bullet$  OBS : Expansion  $\rightarrow$  simple and explicit opt. stop. rule
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 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

Intuition + Main results ○○○○○○○● Future Research

- THEOREM : Solving opt.stop equal to solving min-cut
- OBS : Novel unification of many duality results for opt.stop
  - Karatzas, Kogan and Haugh, Rogers, Glasserman, ...
  - Opt. dual martingale  $\leftrightarrow$  max-flow
- OBS : Simple proofs and intuition about many past results
  - Tree network  $\rightarrow$  greedy works, block. flow is opt, ...
- OBS : Our algorithms can be interpreted as ...
   Fast rand, iter, method for max-flow on massive tree
  - Total flow pushed in round kits Elminiet 7 Z<sup>A</sup>
- $\bullet$  OBS : Expansion  $\rightarrow$  simple and explicit opt. dual sol.
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- OBS : Expansion  $\rightarrow$  simple and explicit opt. dual sol.
- $\bullet$  OBS : Expansion  $\rightarrow$  simple and explicit opt. stop. rule
  - Stop when  $Z_t = E\left[\sum_{k=1}^{\infty} \min_{i \in [1,T]} Z_i^k | \mathcal{F}_t\right]$
  - Stop when you reach sat. edge (min-cut)

 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

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- $\bullet~\text{OBS}:\text{Expansion}\rightarrow\text{simple}~\text{and}~\text{explicit}~\text{opt.}$  stop. rule
  - Stop when  $Z_t = E\left[\sum_{k=1}^{\infty} \min_{i \in [1,T]} Z_i^k | \mathcal{F}_t\right]$
  - Stop when you reach sat. edge (min-cut)

 $\begin{array}{l} \textbf{Model} + \textbf{Problem} \\ \textbf{000} \end{array}$ 

Intuition + Main results ○○○○○○○○● Future Research

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- THEOREM : Solving opt.stop equal to solving min-cut
- OBS : Novel unification of many duality results for opt.stop
  - Karatzas, Kogan and Haugh, Rogers, Glasserman, ...
  - Opt. dual martingale  $\leftrightarrow$  max-flow
- OBS : Simple proofs and intuition about many past results
  - Tree network  $\rightarrow$  greedy works, block. flow is opt, ...
- OBS : Our algorithms can be interpreted as ....
  - Fast rand. iter. method for max-flow on massive tree
  - Amount pushed on a given edge in round k is ...
    - Explicit (normed) cond. exp.
  - Total flow pushed in round k is  $E[\min_{t \in [1,T]} Z_t^k]$
- $\bullet~\text{OBS}$  : Expansion  $\rightarrow$  simple and explicit opt. dual sol.
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Punchline	$\begin{array}{c} \textbf{Model} + \textbf{Problem} \\ \text{ooo} \end{array}$	Intuition + Main results	Future Research ●০০০
Outline			



2 Model + Problem

Intuition + Main results



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Intuition + Main results

Future Research

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# **Future research**

#### Fast implementation and comparison to past approaches

- Glasserman, Longstaff-Schwartz, Andersen and Broadie, Belomestny, Schoenmakers, Bender, Christensen, Ibanez, Jamshidian, Farias, Kohler, Lelong, ...
- Especially on real financial data and problems
- Produce and share a usable code

 $\underset{000}{\text{Model}} + \text{Problem}$ 

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# Future research cont.

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- Lower bounds, comp. complexity, randomization
- Modified / different such expansions
- Other tools from network flows
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Intuition + Main results

Future Research

#### Extra credit : fast convergence proof

• Recall : 
$$Z_t^k = Z_t^{k-1} - E[\min_{i \in [1,T]} Z_i^{k-1} | \mathcal{F}_t]$$

• Claim :  $Z_t^k \ge 0$  for all t, k

• Claim :  $\{Z_t^k, k \ge 1\}$  is  $\downarrow$  for all t

- Claim :  $Z_T^k = Z_T \sum_{i=1}^{k-1} \min_{t \in [1,T]} Z_t^i$  for all k
- $\rightarrow Z_T (k-1) \times \min_{t \in [1,T]} Z_t^{k-1} \ge 0$  w.p.1

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$$\rightarrow \min_{t \in [1,T]} Z_t^{k-1} \leq \frac{U}{k-1}$$
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•  $\rightarrow \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^k] \leq \frac{U}{k}$ 

- But OPT =  $\sum_{i=1}^{k} E[\min_{t \in [1,T]} Z_t^i] + \inf_{\tau \in T} E[Z_{\tau}^{k+1}] \quad \forall k$
- THEOREM :  $\left| \text{OPT} \sum_{i=1}^{k} E[\min_{t \in [1,T]} Z_t^i] \right| \le \frac{U}{k+1}$
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- Claim :  $Z_t^k \ge 0$  for all t, k
- Claim :  $\{Z_t^k, k \ge 1\}$  is  $\downarrow$  for all t
- Claim :  $Z_T^k = Z_T \sum_{i=1}^{k-1} \min_{t \in [1,T]} Z_t^i$  for all k
- $\rightarrow Z_T (k-1) \times \min_{t \in [1,T]} Z_t^{k-1} \ge 0$  w.p.1

• 
$$\rightarrow \min_{t \in [1,T]} Z_t^{k-1} \leq \frac{U}{k-1}$$
 w.p.1

- $\rightarrow \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^k] \leq \frac{U}{k}$
- But OPT =  $\sum_{i=1}^{k} E[\min_{t \in [1,T]} Z_t^i] + \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^{k+1}] \quad \forall k$
- THEOREM :  $\left| \text{OPT} \sum_{i=1}^{k} E[\min_{t \in [1,T]} Z_t^i] \right| \le \frac{U}{k+1}$
- Note : Also prove other bounds ind. of U (even if  $U=\infty$ )

 $\underset{000}{\text{Model}} + \text{Problem}$ 

Intuition + Main results

Future Research

#### Extra credit : fast convergence proof

• Recall : 
$$Z_t^k = Z_t^{k-1} - E[\min_{i \in [1,T]} Z_i^{k-1} | \mathcal{F}_t]$$

• Claim : 
$$Z_t^k \ge 0$$
 for all  $t, k$ 

• Claim : 
$$\{Z_t^k, k \ge 1\}$$
 is  $\downarrow$  for all  $t$ 

• Claim : 
$$Z_T^k = Z_T - \sum_{i=1}^{k-1} \min_{t \in [1,T]} Z_t^i$$
 for all k

• 
$$\rightarrow Z_T - (k-1) \times \min_{t \in [1,T]} Z_t^{k-1} \ge 0$$
 w.p.1

• 
$$\rightarrow \min_{t \in [1,T]} Z_t^{k-1} \leq \frac{U}{k-1}$$
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• 
$$\rightarrow \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^k] \leq \frac{U}{k}$$

• But OPT = 
$$\sum_{i=1}^{k} E[\min_{t \in [1,T]} Z_t^i] + \inf_{\tau \in \mathcal{T}} E[Z_{\tau}^{k+1}] \quad \forall k$$

• THEOREM : 
$$\left| \text{OPT} - \sum_{i=1}^{k} E[\min_{t \in [1,T]} Z_{t}^{i}] \right| \leq \frac{U}{k+1}$$

• Note : Also prove other bounds ind. of U (even if  $U = \infty$ )

 $\begin{array}{c} \textbf{Model} + \textbf{Problem} \\ \circ \circ \circ \end{array}$ 

Intuition + Main results

Future Research

#### Extra credit : fast convergence proof

• Recall : 
$$Z_t^k = Z_t^{k-1} - E[\min_{i \in [1,T]} Z_i^{k-1} | \mathcal{F}_t]$$

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• Note : Also prove other bounds ind. of U (even if  $U = \infty$ )