Lecture 1: Mechanism design for Fair Division

The first task of a microeconomic approach to Fair Division is prescriptive: what definition of Fairness is compatible with the other paramount normative concern of Efficiency (Pareto optimality)? Next, to implement a fair and efficient division of the manna, we must take into account the descriptive constraints of Incentive Compatibility.

We discuss two allocation problems where these three requirements are "miraculously" compatible, in sharp contrast to the general impossibility results discvovered in the first three decades of the mechanism design literature.

Problem 1: One dimensional manna The manna ω is a positive amount of some non disposable item (the entire manna must be distributed): hours of baby-sitting between family members, shares of a stock between investors, teaching loads between teachers, etc.. Each agent i has single-peaked (i. e., convex) preferences over his/her share z_i of the manna, and an optimal consumption level π_i.

The canonical mechanism introduced in Sprumont (1991) is called the *uniform division rule.* It starts from the equal split (fair share) allocation $\overline{z}_i = \frac{1}{n}\omega$ for all i, and stays there if the peaks π_i are all on the same side of the fair share. If this is no the case write N^{under} and N^{over} for, repectively, the non empty sets of *underdemanding* agents, $\pi_i < \frac{1}{n}\omega$, and of *overdemanding* agents,$\pi_j > \frac{1}{n}\omega$; and N^{fs} is the set of agents k whose peak is "just right": $\pi_k = \frac{1}{n}\omega$.

Each agent $k \in N^{fs}$ receives his fair share $z_k = \frac{1}{n}\omega$. If the manna is overdemanded, each $i \in N^{under}$ gets her peak, $z_i = \pi_i$, while agents j in N^{over} get weakly less than their peak (they are rationed). Then the rule equalizes the gains $(z_j - \frac{1}{n}\omega)$ for the latter agents, as much as allowed by the efficiency constraint $z_j \leq \pi_j$. If the manna is underdemanded, agents in $N^{over} \cup N^{fs}$ get their peak while those in N^{under} now get weakly more than their peak, and the gains $(\frac{1}{n}\omega - z_j)$ are similarly equalized.

Theorem (Sprumont (1991), Ching (1994)): *The uniform rule combines simplicity of individual messages, with Efficiency, Fairness in the sense of the Envy Free test, and the strong form of incentive compatibility called GroupStrategyproofness. The latter properties uniquely characterize the rule.*

Moulin (2017) shows that these result holds in all problems where individual allocations are single dimensional and preferences are single-peaked, provided the feasibility constraints are convex. Voting over an interval is an example.

Problem 2 Complementary goods Now the manna ω contains several goods that are perfect complements: each agent "needs" all goods to generate utility from the manna. For instance entrepreneurs share a manna made of capital goods, raw materials and a labor force, each needs all three to open shop but not necessarily in the same proportions. In cloud computing, each user needs a personal combination of memory, computing resources and bandwidth to perform his task (Ghodsi et al. (2011)). And so on.

Formally, A is the set of goods, and agent i's utility function takes the form:

$$u_i(z_i) = \min_{a \in A}\{\frac{z_{ia}}{w_{ia}}\}$$

The rule of interest in this problem is the Egalitarian Equivalent rule EE(ω) due to Pazner and Schmeidler (1979). It looks for the largest parameter λ such that for some feasible allocation z and all i we have

$$u_i(z_i) \geq u_i(\lambda\theta)$$

and implements the allocation z.

Theorem (Ghodsi et al. (2011)): *The division rulke EE(ω) with disposal of redundant goods is Efficient, GroupStrategyproof, and picks an Envy Free allocation.*

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Lecture 2: Fair Division on the Internet

We now assume that the manna consists of perfect substitutes goods, so that preferences are represented by additive utilities. Such preferences are realistic when we divide the manna into truly "unrelated" goods such as a computer, a bicycle and a portrait in the family heirlooms, where the pair of matching chandeliers must be counted as one item. The practicality of eliciting additive utilities is illustrated by the success of SPLIDDIT (www.spliddit.org/) designed by Goldman & Procaccia (2014), a user-friendly platform where users report additive utilities by dividing 1000 points over the items.

In the additive domain, the prominent division rule is the Competitive one, obtained by endowing all agents with an equal virtuel budget, and finding the unique price at which competitive demands clear the manna. Interestingly, this rule is deeply connected to a well known solution of the general bargaining problem.

Theorem *The two following statements are equivalent:*

i) The feasible allocation z is competitive;

ii) The corresponding utility profile maximizes the Nash product over all feasible profiles.

iii) The Competitive allocations all have the same welfare, and the same price.

We discuss the consequences of this result and its limits, when the manna consists of undesirable bads, or when the goods are indivisible. In turn this suggests several open questions for future research.

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