About Allocation Problems and Equilibrium Bidding Strategies in Procurement Auctions

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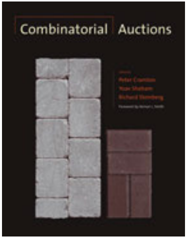
Industrial Procurement Auctions



- Multi-object auctions are ubiquitous in industrial procurement and transportation.
- Combinatorial auctions are regularly used to account for economies of scale and scope (round routes in transportation, volume discounts, etc.)

Economies of Scale

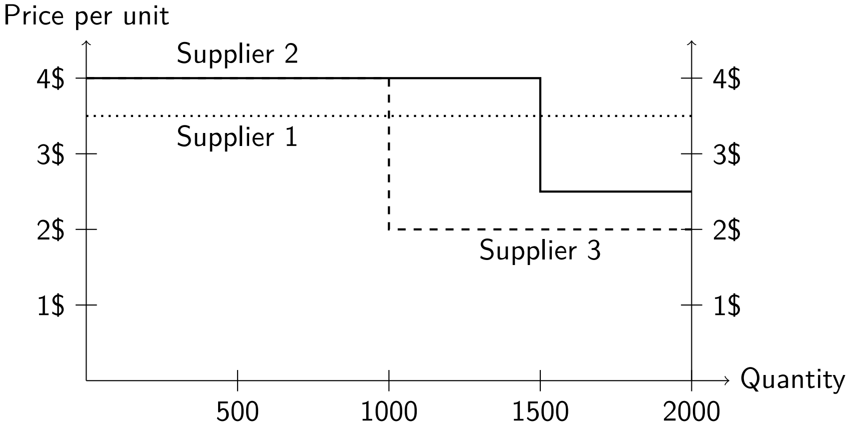
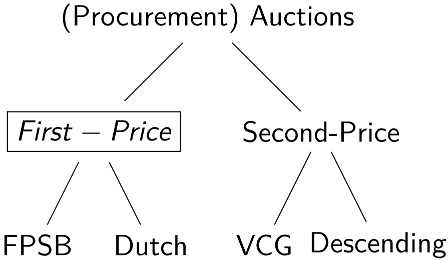


Figure: Example of volume discounts on a single item

First-Price Auctions are Wide-Spread



- VCG ist the unique DSIC mechanism, but it is rarely used in the field.
- Equilibrium bidding strategies of single-object first-price auctions are complex and rarely predictive (e.g., overbidding puzzle).
- Multi-object first-price auctions ??

In a Nutshell ...

- Split-award auctions are frequently used in industrial procurement and allow for bids on packages of objects
- Almost no literature on such first-price combinatorial auctions
  - Bernheim and Whinston (1986), Anton and Yao (QJE, 1992)
- Our analysis
  - Dutch and Dutch-FPSB split-award auctions for $n \geq 2$ suppliers

Main Findings

- Theory: Strategic differences between three first-price formats.
- The Dutch auction only has efficient equilibria, in contrast to the FPSB auction.
- Equilibrium strategies explain the experimental data surprisingly well.

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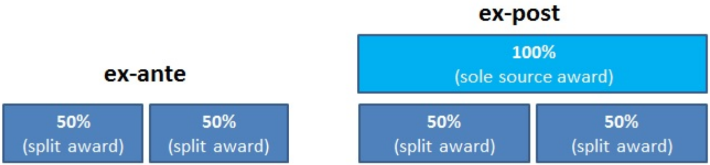
Ex-ante vs. Ex-post Split-Award Auctions

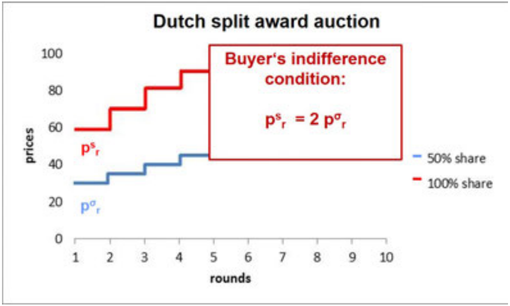
- Split-award auctions are a wide-spread form of combinatorial auctions in procurement.



split-award auctions	**ex-ante**	**ex-post**
bids	p_i^σ $(i = 1, .., n)$	p_i^σ and p_i^s $(i = 1, .., n)$
sourcing strategy	dual sourcing	dual vs. single sourcing
buyer's price in FPSB	$\min_{i \neq j}\{p_i^\sigma + p_j^\sigma\}$	$\min_{i \neq j}\{p_i^\sigma + p_j^\sigma\}$ vs. $\min_i\{p_i^s\}$

Dutch and Dutch-FPSB - Period 1

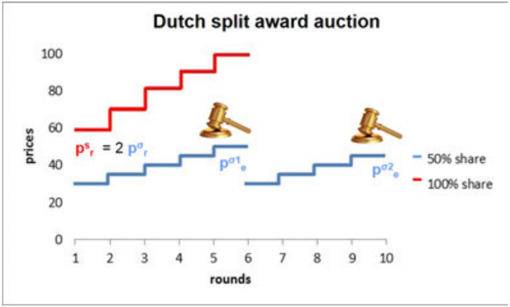
- Ascending (reverse) auction with offers for 50% and 100% in each round.



- Bidders can either accept 50% ($\rightarrow$ continuation with remaining 50%), accept 100% ($\rightarrow$ immediate termination) or reject both.

Dutch and Dutch-FPSB - Period 2

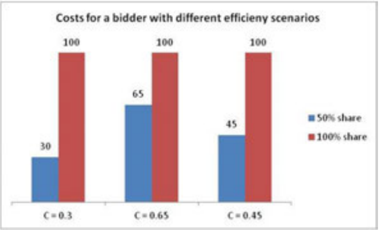
- **Dutch**: After accepting an offer for 50%, the remaining 50% is offered to all bidders for the initial starting price.



- **Dutch-FPSB**: Both bidders submit sealed bids in stage 2. In case of a tie in stage 2, the split is selected.

Model Assumptions

- $n \geq 2$ ex-ante symmetric, risk-neutral and profit-maximizing suppliers
- Share sizes: 100% or 50% (wlog.) of the total demand
- IPV setting: $\Theta_i \sim F(\cdot)$ over $[\underline{\Theta}, \overline{\Theta}]$
- **Dual Source Efficiency (DSE):** $C < \frac{\Theta}{\underline{\Theta}+\overline{\Theta}} < 0.5$
- In DSE, the efficient split outcome requires coordination, and the winner-takes-all (WTA) outcome is inefficient!
- C symmetric and publicly known among suppliers



- **Costs for 50%**: $k^{\sigma}(\Theta_i) = \Theta_i C$
- **Costs for 100%**: $k^{s}(\Theta_i) = \Theta_i$

WTA Equilibrium in FPSB with $n \geq 2$ Bidders

Theorem

For a bidder i, the following bidding strategy $S_i = (p_e^s(\Theta_i), p_e^\sigma(\Theta_i))$ is a ***WTA*** **equilibrium** *in the* ***FPSB split award auction****:*

$$p_e^s(\Theta_i) = \Theta_i + \frac{\int_{\Theta_i}^{\overline{\Theta}} (1 - F(t))^{n-1} dt}{(1 - F(\Theta_i))^{n-1}} \text{ and}$$

$$p_e^\sigma(\Theta_i)^* = p_e^s(\Theta_i)$$

* Example of a simple price function excluding the split.

FPSB WTA Equilibrium: Proof Sketch

1. *Sole source deviation*:
   - The price $p_e^s(\Theta_i)$ maximizes the expected payoff for winning 100%:

$$\boldsymbol{E[\Pi^s(\Theta_i)] = (p^s(\Theta_i) - \Theta_i)P(p^s(\Theta_i) \leq p_e^s(\Theta_{1:n-1}))}$$

   - Standard approach (FOC, solving differential equation).
2. *Split deviation*:
   - The split award is excluded by **bid-to-lose prices** in equilibrium.

Shares	Bidder A	Bidder B	Bidder C	Bidder D
50%	130	35	105	150
100%	130	110	105	150

σ Equilibrium in FPSB with $n = 2$ Bidders

Theorem

There are different efficient σ equilibria $(p_e^\sigma, p_e^s(\cdot))$ with $p_e^\sigma \in [\overline{\Theta}C, (1-C)\underline{\Theta}]$, if

$$2p_e^\sigma \leq p_e^s(\Theta) \leq G(\Theta, p_e^\sigma) = p_e^\sigma + \frac{p_e^\sigma - C\overline{\Theta}F(\Theta)}{1-F(\Theta)} \text{ for all } \Theta \in [\underline{\Theta}, \overline{\Theta}]$$

applies.

- The p_e^σ is a *constant bid price* on which both bidders need to coordinate.
- $G(\Theta, p_e^\sigma)$ is an upper bound on the package bid. If the package bid was too high, the opponent could try to undercut the package bid with higher payoff.

σ Equilibrium in Dutch with $n = 2$ Bidders

Theorem

In the Dutch split-award auction model with $n = 2$ risk-neutral bidders with DSE, there is no WTA equilibrium.

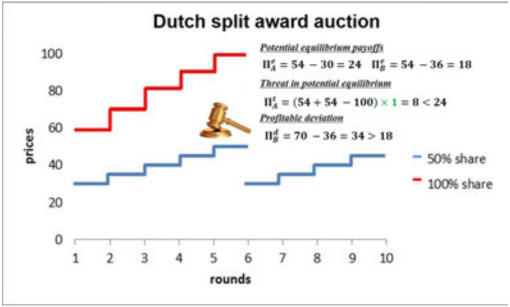
- There is always a round r, where $\overline{\Theta}$ has a higher split payoff in DSE.

Theorem

There is a unique and efficient σ equilibrium with $p_e^\sigma = (1 - C)\underline{\Theta}$ for both bidders. The winner of Period 1 with cost type Θ_A threatens to accept the remaining share for a price of $p_e^{\sigma 2}(\Theta_A) = \Theta_A(1 - C)$.

Dutch Split-Award Auction with 2 Bidders

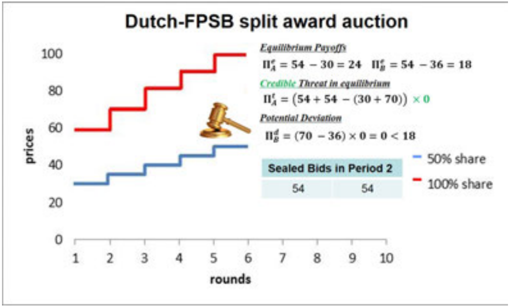
- Example: $\Theta \sim U[100, 140]$, $C = 0.3$, 2 bidders $\Theta_A = 100$, $\Theta_B = 120$
- Dutch: $p_e^\sigma = 70$ vs. Dutch-FPSB: $p_e^\sigma \in [54, 70]$



- Dutch: Threat must be carried out in case of opponent's deviation.
- Threat only credible if payoff not lower than in Period 1.

Dutch-FPSB with 2 Bidders

- Example: $\Theta \sim U[100, 140]$, $C = 0.3$, 2 bidders $\Theta_A = 100$, $\Theta_B = 120$
- Dutch: $p_e^\sigma = 70$ vs. Dutch-FPSB: $p_e^\sigma \in [54, 70]$



- Dutch-FPSB: Threat credible as potential deviations from equilibrium become unprofitable.

σ Equilibrium in Dutch-FPSB with $n = 2$ Bidders

Theorem

In the Dutch-FPSB split-award auction model with $n = 2$ risk-neutral bidders with DSE, there is no WTA equilibrium.

Theorem

There are different efficient σ equilibria with $p_e^\sigma \in [\overline{\Theta}C, (1-C)\underline{\Theta}]$, if

$$\Theta \leq G(\Theta, p_e^\sigma) = p_e^\sigma + \frac{p_e^\sigma - C\overline{\Theta}F(\Theta)}{1 - F(\Theta)} \text{ for all } \Theta \in [\underline{\Theta}, \overline{\Theta}]$$

applies. The winner of Period 1 with cost type Θ_A threatens to accept the remaining share at a price of $p_e^{\sigma 2}(\Theta_A) = p_e^\sigma$.

Welfare Analysis (Summary)

- $n = 2$
  - The FPSB split-award auction exhibits an equilibrium selection problem.
  - Dutch and Dutch-FPSB have only efficient equilibria.
  - There is cost equivalence, only if the σ equilibrium is selected in the FPSB auction.
- $n > 2$
  - In theory there is still an equilibrium selection problem in the FPSB auction.
  - There are no pooling prices anymore and σ equilibria depend on the cost type.

Experimental Design

- Experimental setting: $\Theta \sim U[100, 140], C = 0.3$, s.t. the σ equilibrium is payoff dominant in the FPSB auction.
- 274 subjects, random rematching within sessions.
- Treatments:
  - 2 bidders: 345 FPSB auctions, 360 Dutch and Dutch-FPSB auctions.
  - 3 bidders: 180 FPSB auctions, 240 Dutch and Dutch-FPSB auctions.

Experimental Results

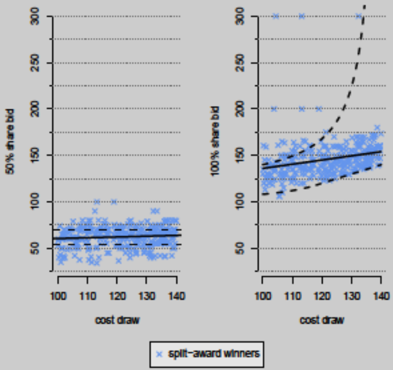
Efficiency and Procurement Costs (Summary)

Setting	2x2		2x3	
	costs	**efficiency**	**costs**	**efficiency**
FPSB	130	45%	76	100%
Dutch	155	64%	79	100%
Dutch - FPSB	130	82%	76	100%

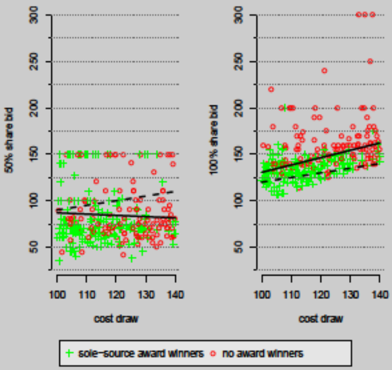
⇒ Dutch-FPSB yields efficiency and low total cost.
⇒ Evidence for pooling.

Bidding Behavior: Two-Bidder FPSB Auction

Split-Award Winners

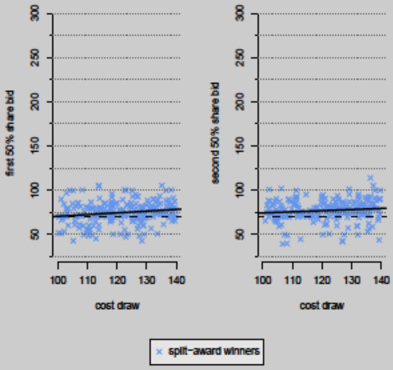


Sole-Source-Award Winners

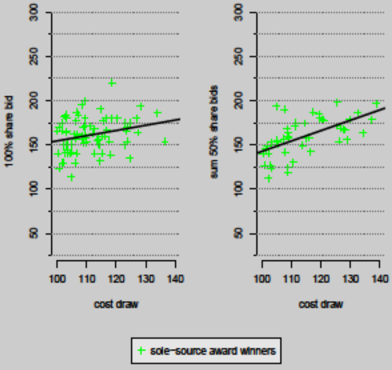


Bidding Behavior: Two-Bidder Dutch Auction

Split-Award Winners

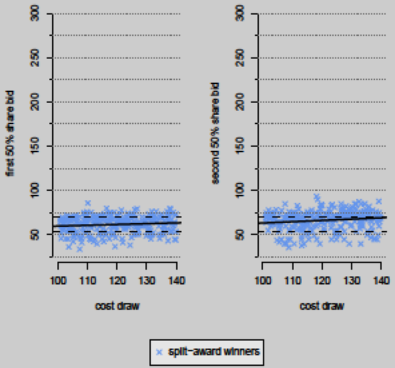


Sole-Source-Award Winners

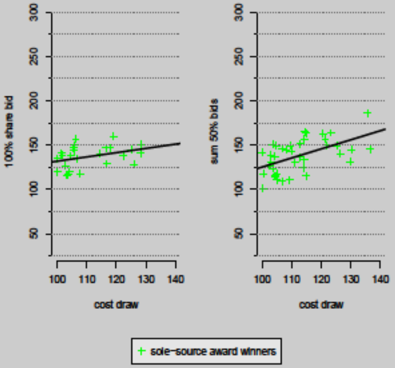


Bidding Behavior: Two-Bidder Dutch-FPSB Auction

Split-Award Winners

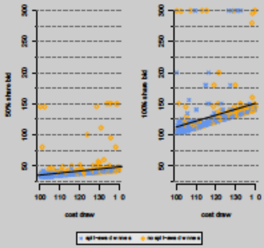


Sole-Source-Award Winners

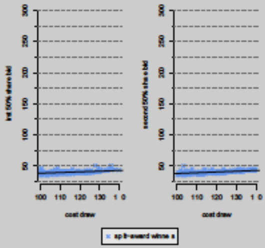


Bidding Behavior: Three-Bidder Auctions

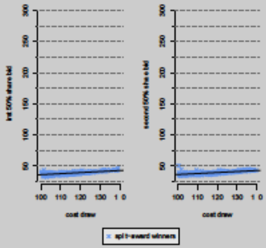
Split-Award Winners FPSB



Split-Award Winners Dutch



Split-Award Winners Dutch-FPSB



Summary

- **Equilibrium Analysis**:
  - The information revealed in the Dutch(-FPSB) helps bidders coordinate on the efficient outcome.
  - There are equilibrium selection problems in the FPSB.
  - In the Dutch(-FPSB) only efficient equilibria exist.
- **Experimental Results**:
  - 2x2 setting: Dutch-FPSB leads to high efficiency and low costs.
  - 2x3 setting: Having a third bidder leads to substantial cost savings.