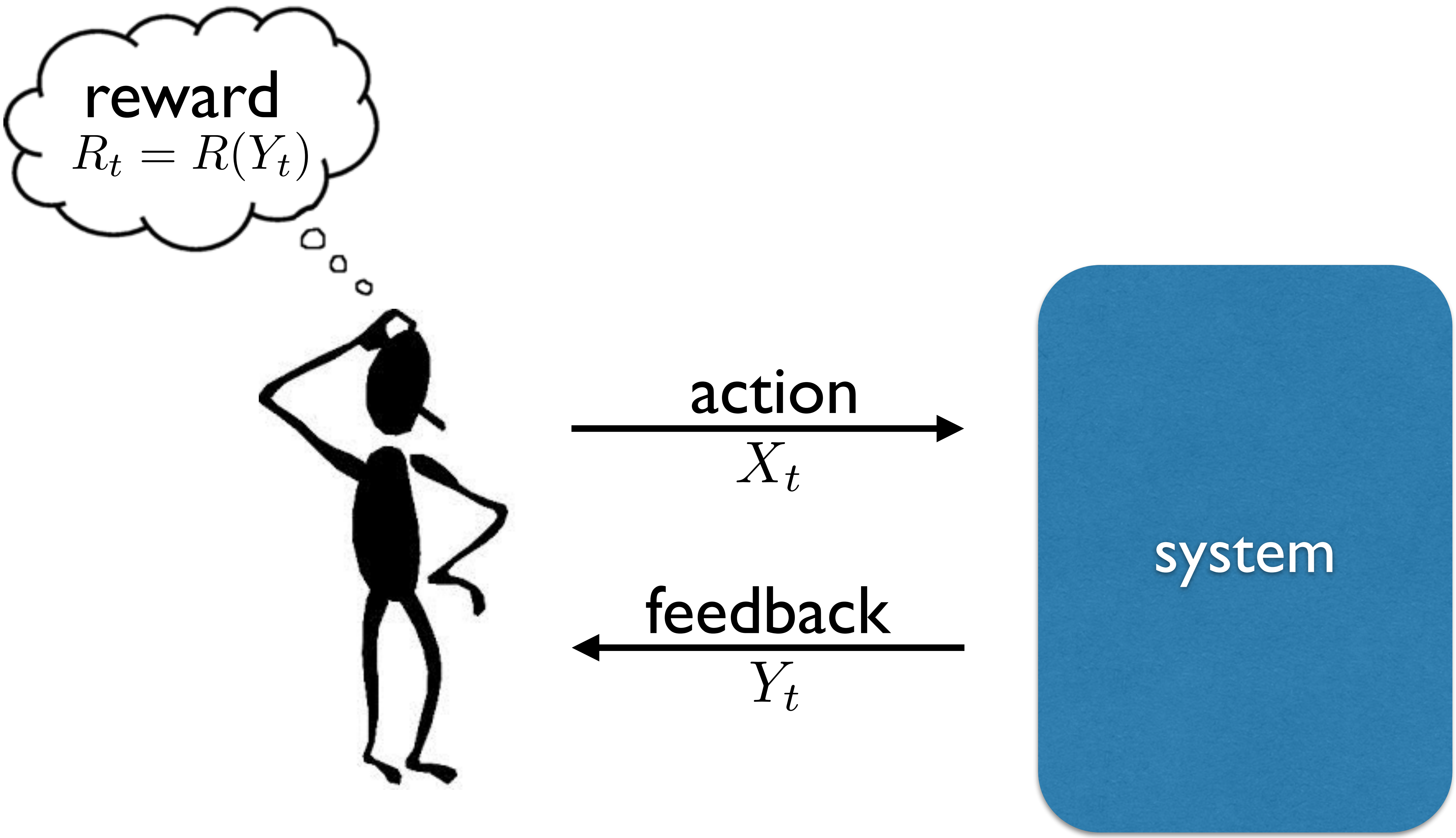
Learning to Optimize

Delayed Consequences

Benjamin Van Roy

work done with Ian Osband, Dan Russo, Zheng Wen

Learning to Optimize



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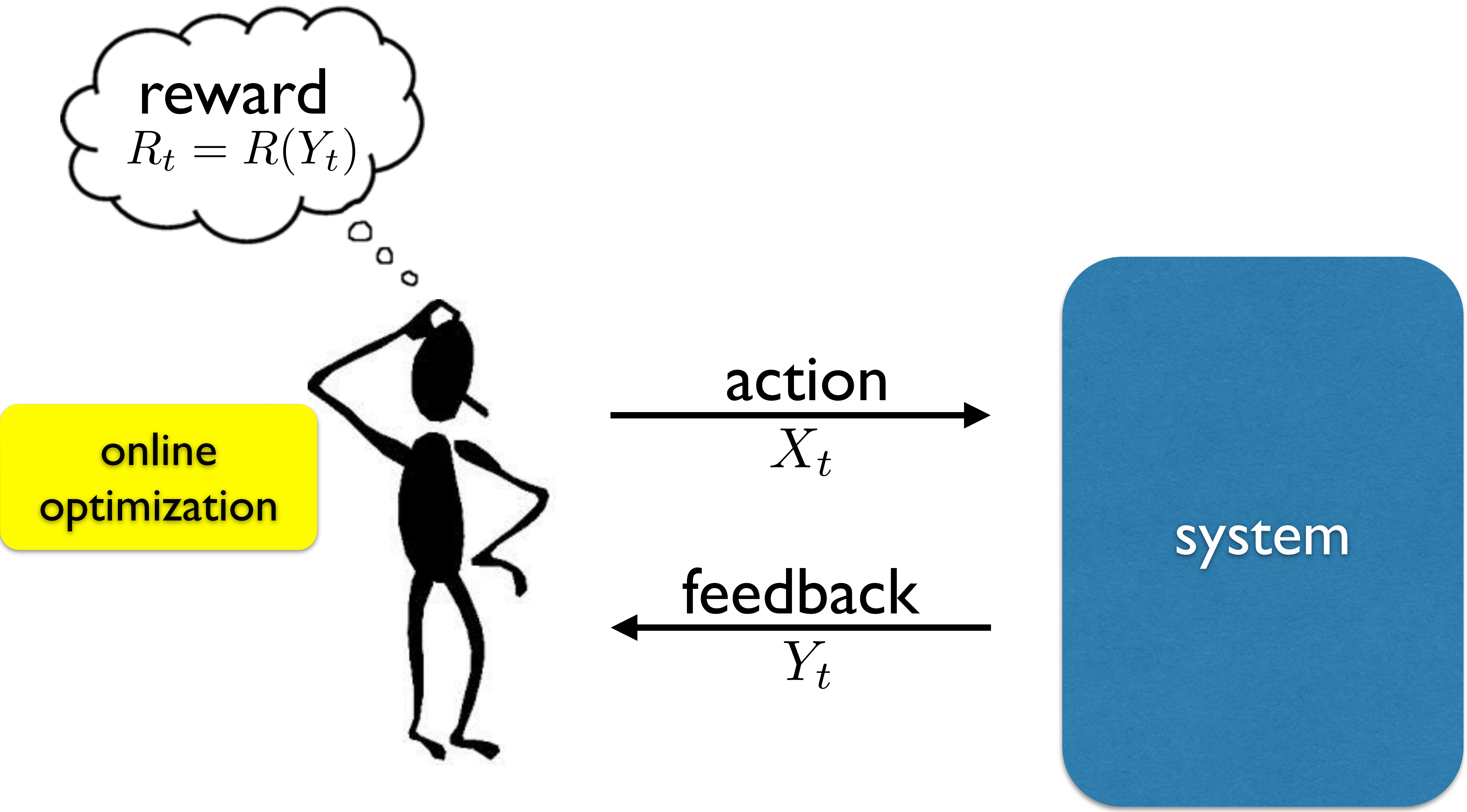

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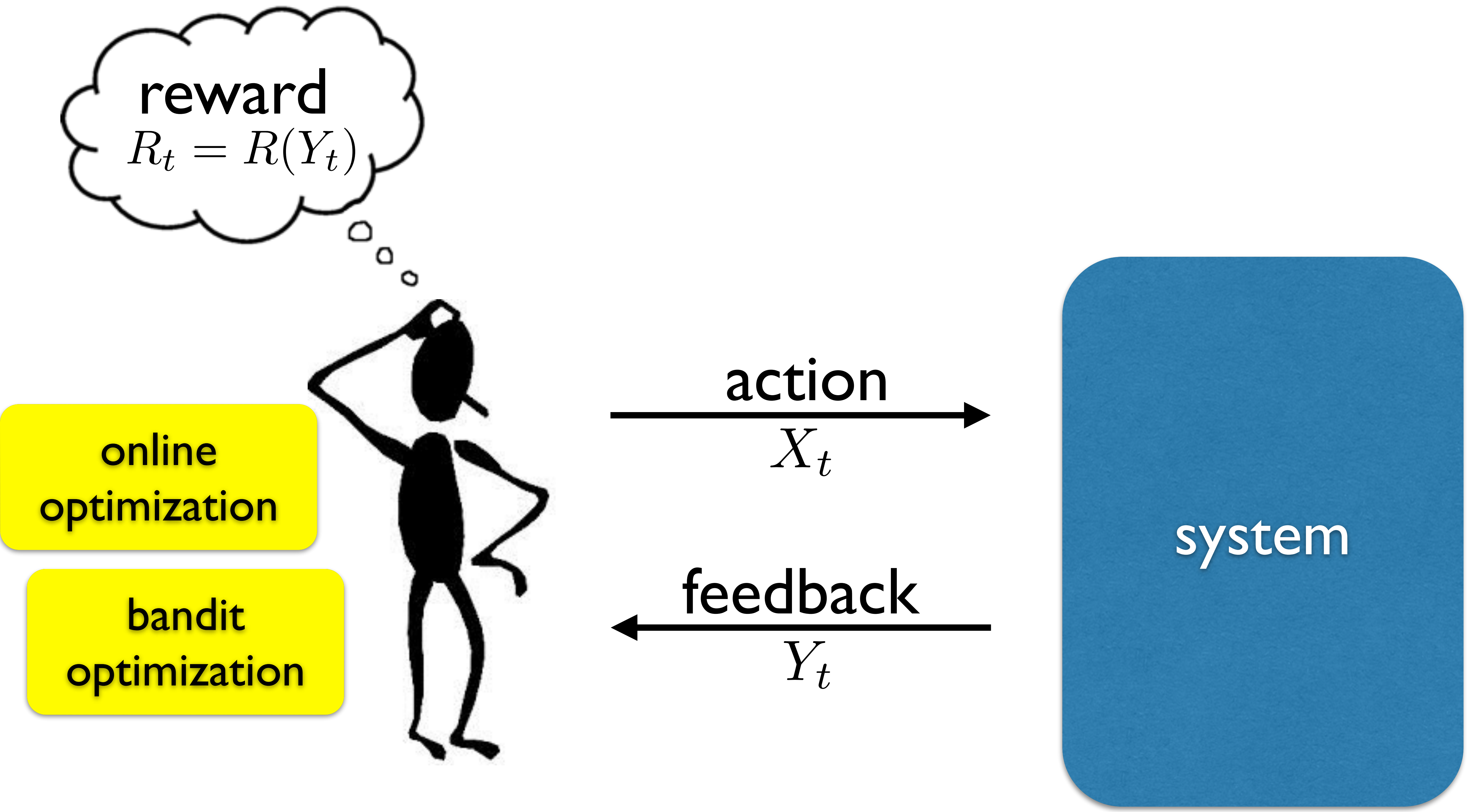

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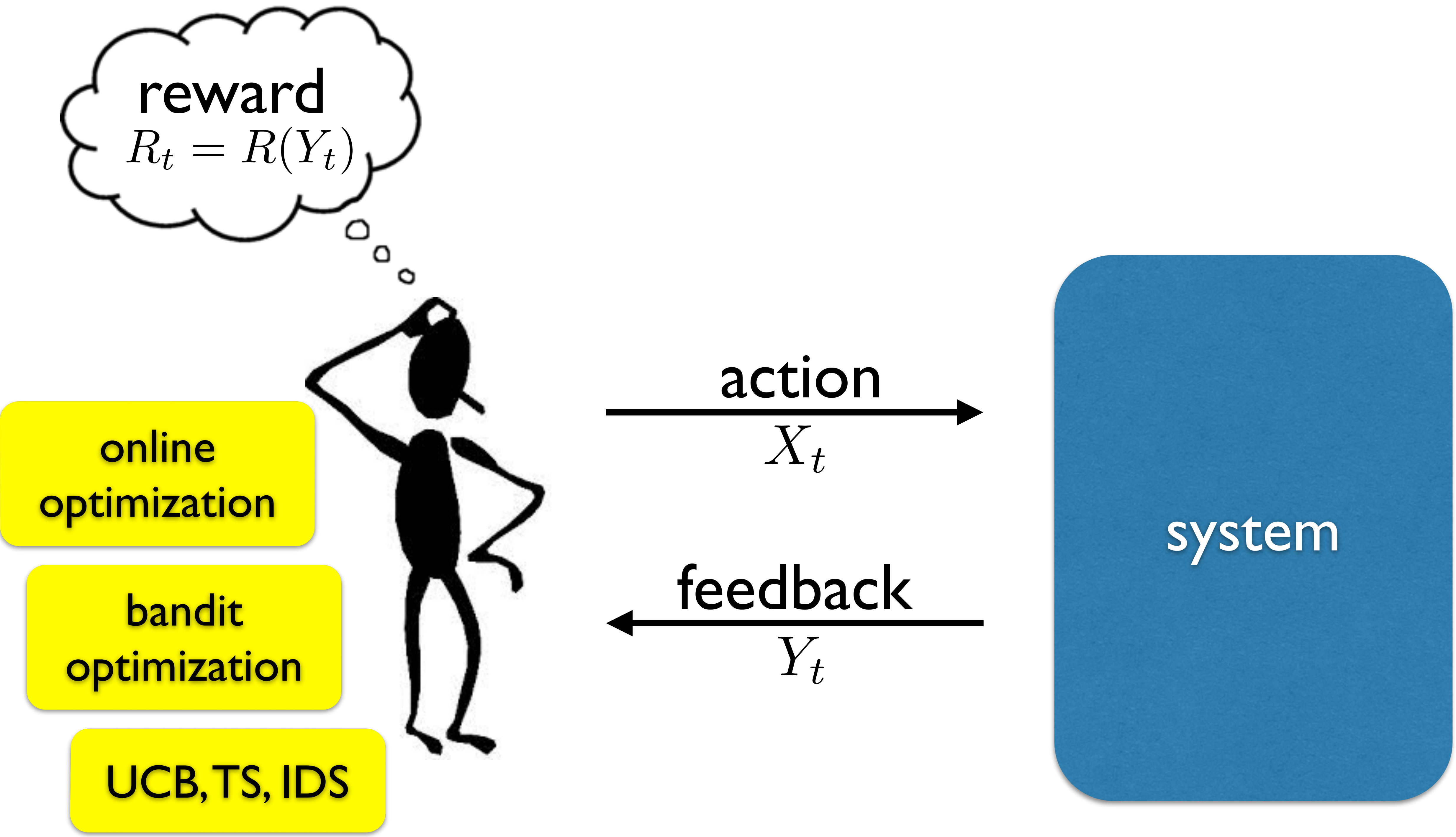

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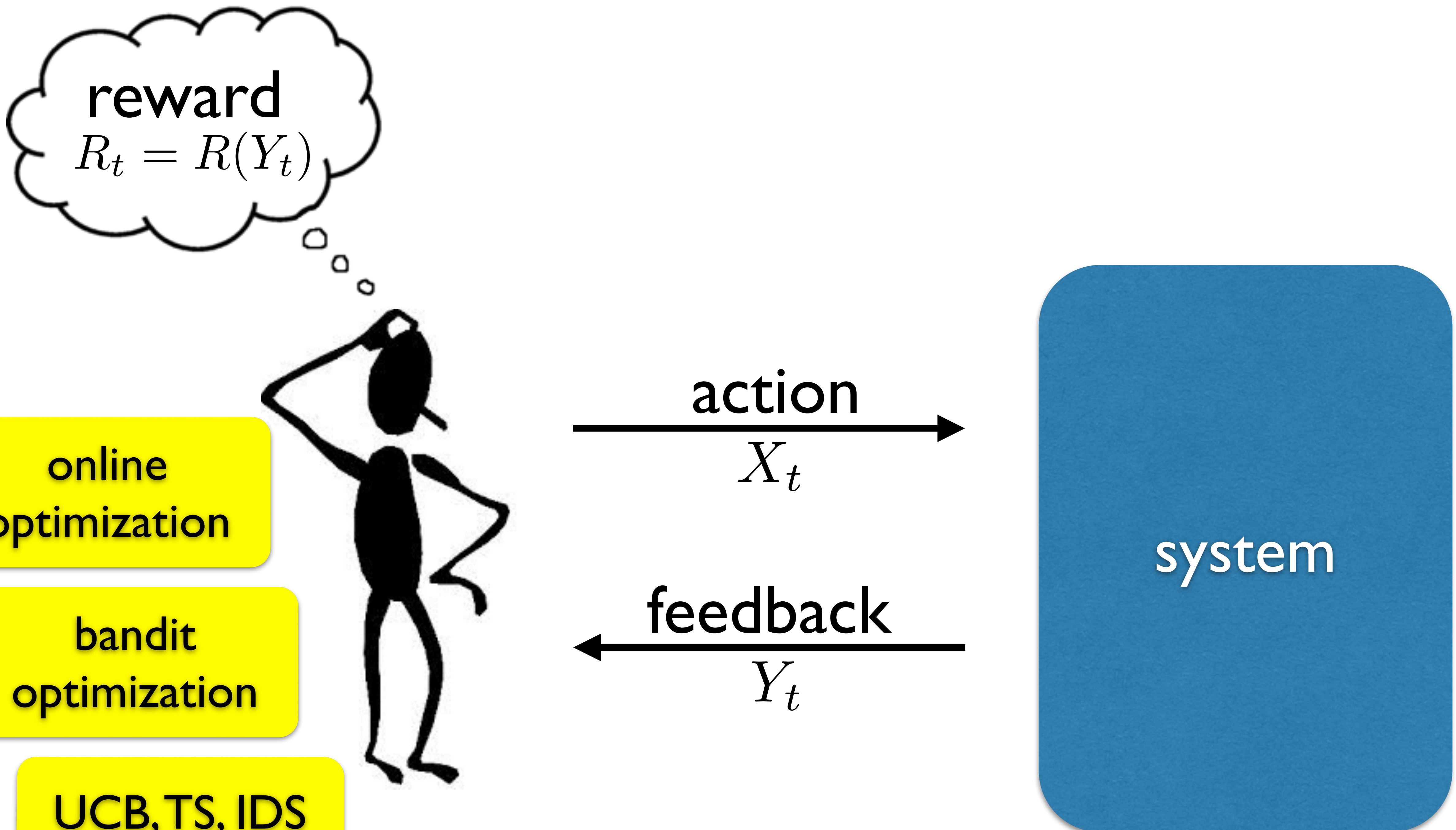

online optimization

bandit optimization

UCB, TS, IDS

delayed consequences

Elements of Statistical Learning

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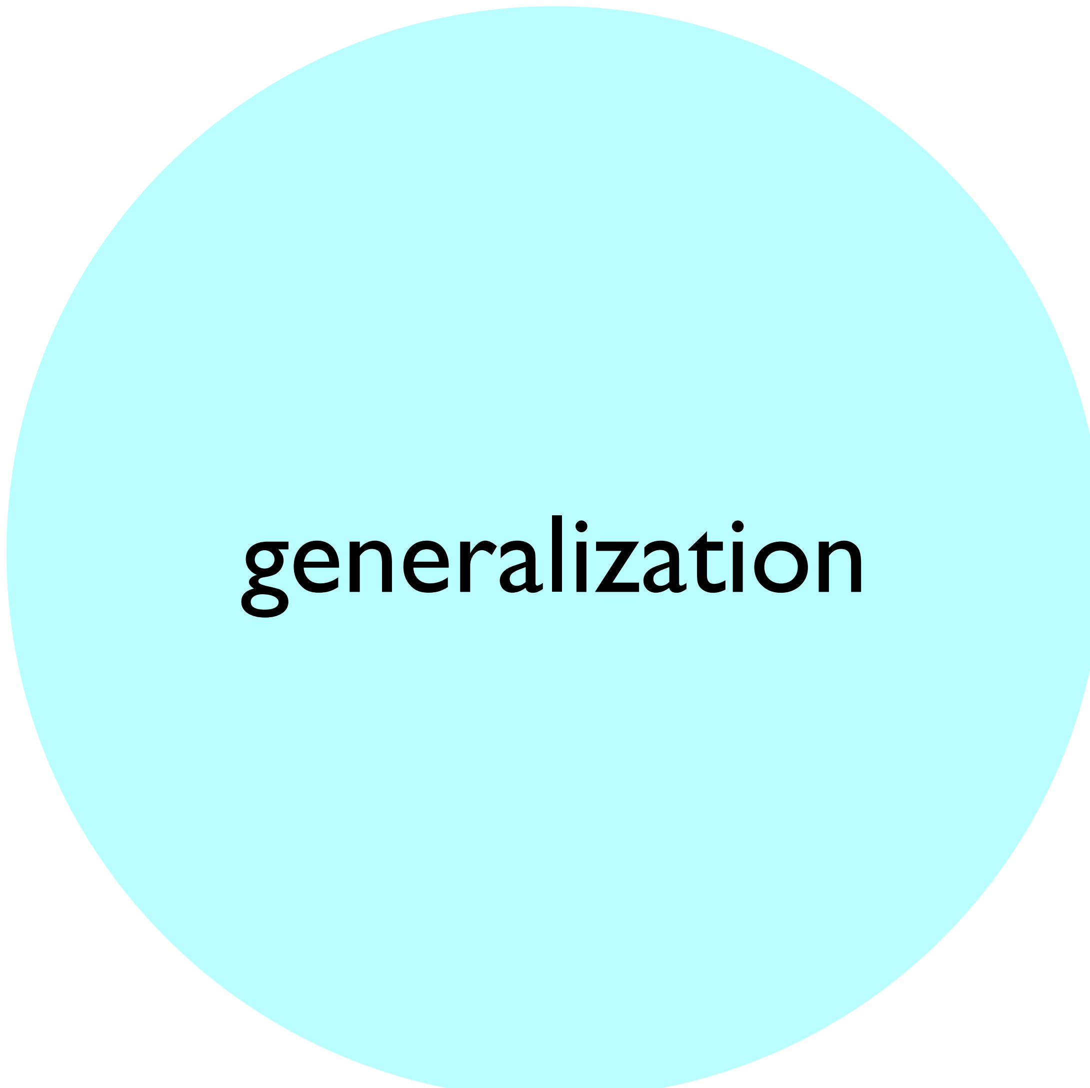

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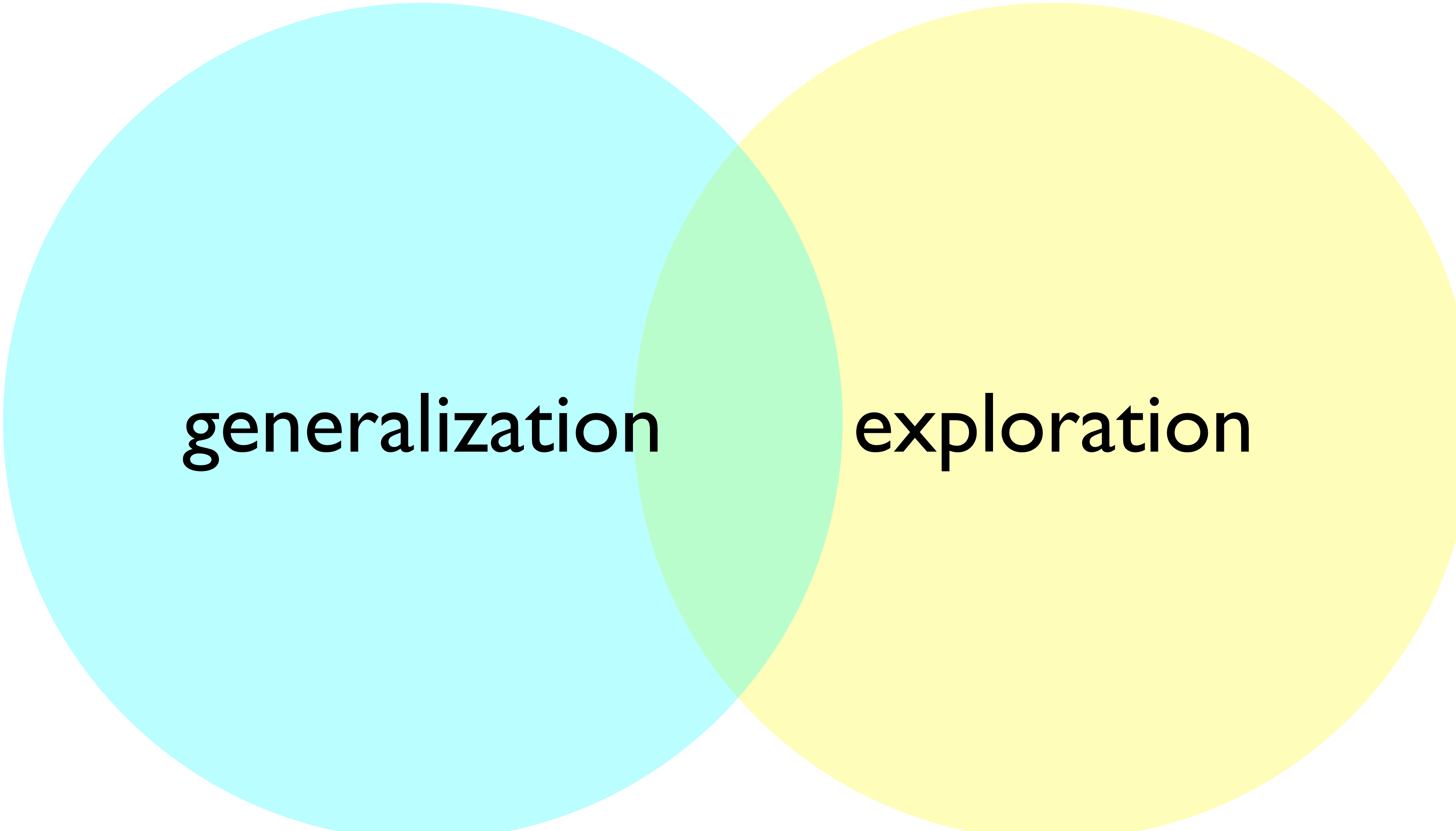

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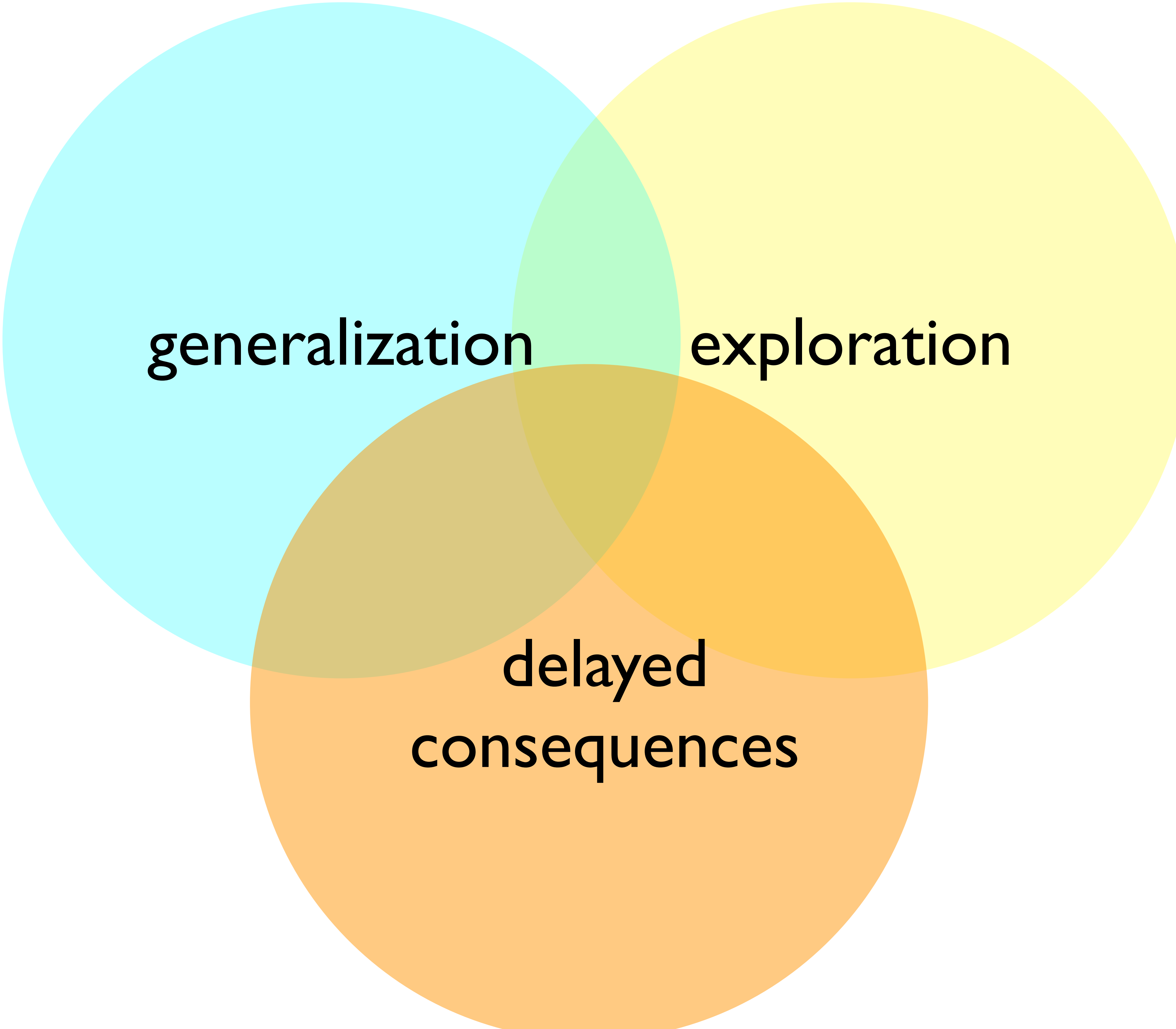

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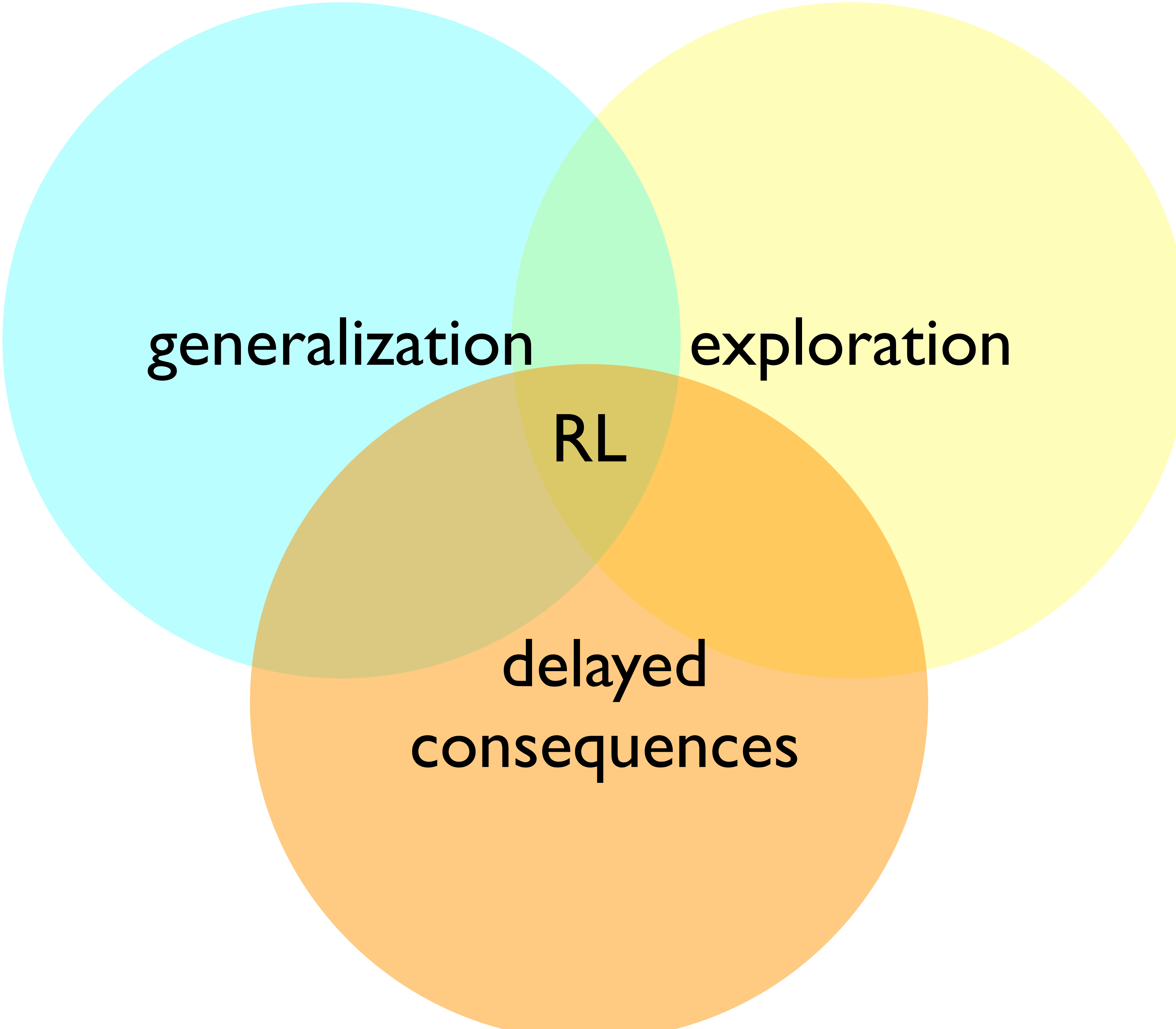
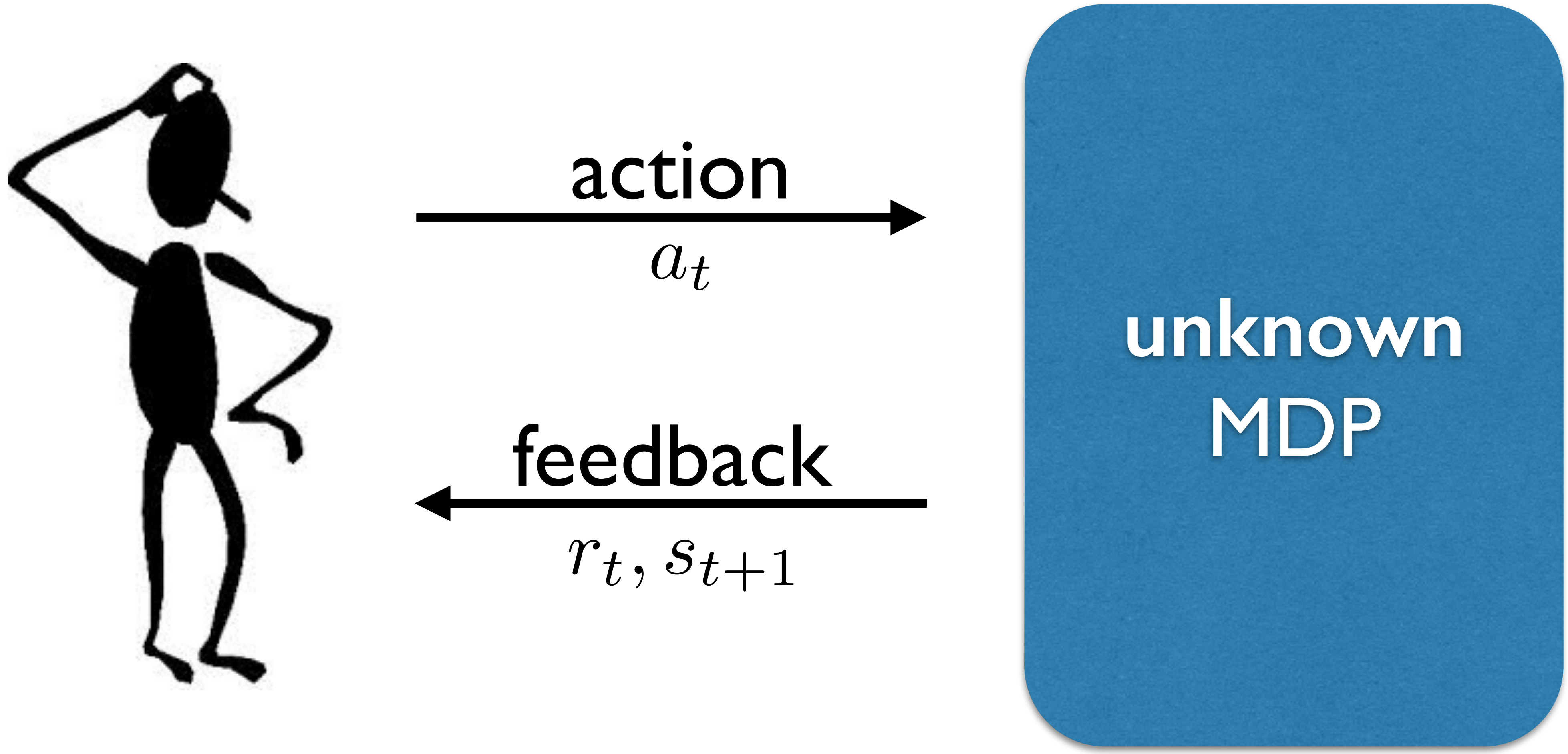


Do delayed consequences matter?

- Robotics
- Web site content optimization
- Online education
- Medical treatments

A Reinforcement Learning Problem



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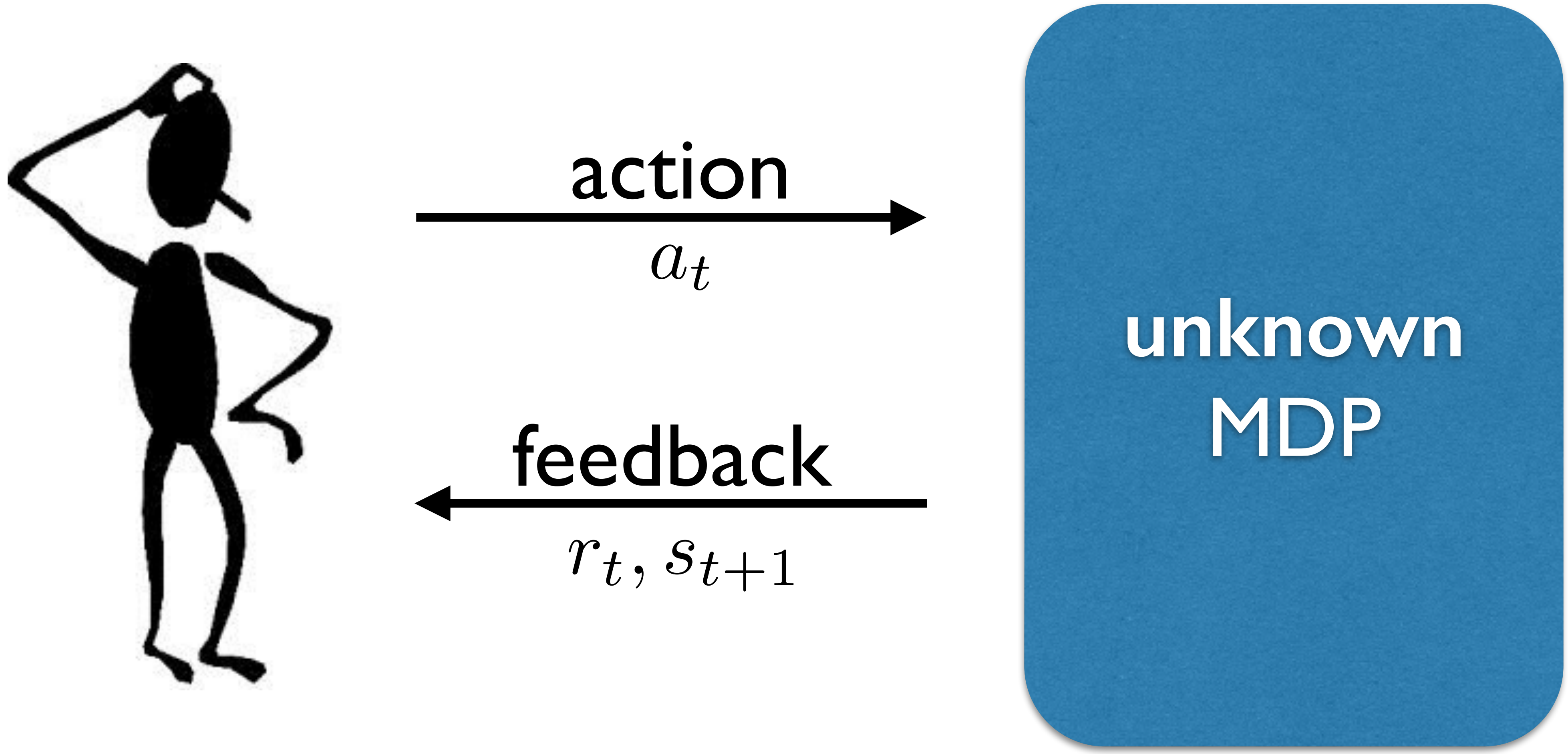

different from the dynamic programming problem

MDP Formulation

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- Unknown MDP
  - Finite horizon, state space, action space
  - Initial state s_0
  - Time-inhomogenous transition and rewards distributions
  - Rewards in [0,1]

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Reinforcement Learning Algorithms

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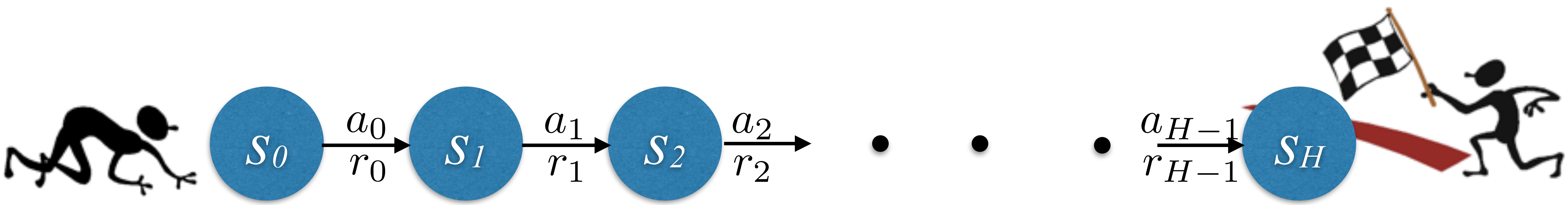
- Episodic learning



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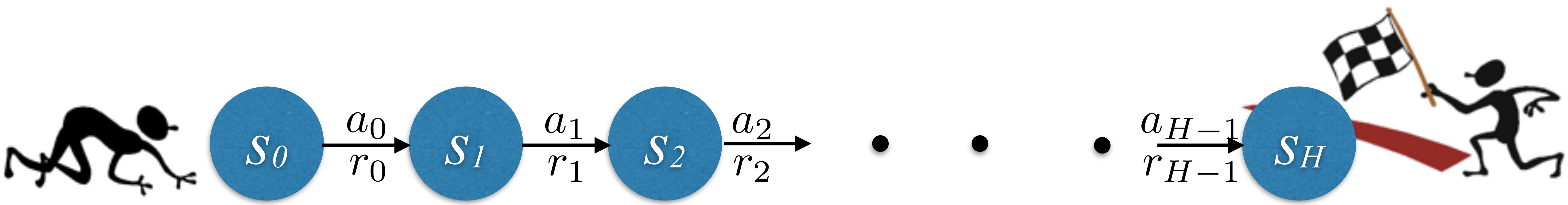

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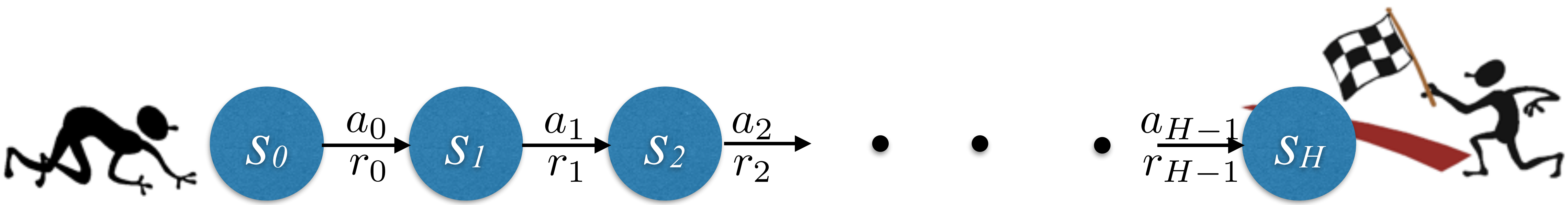

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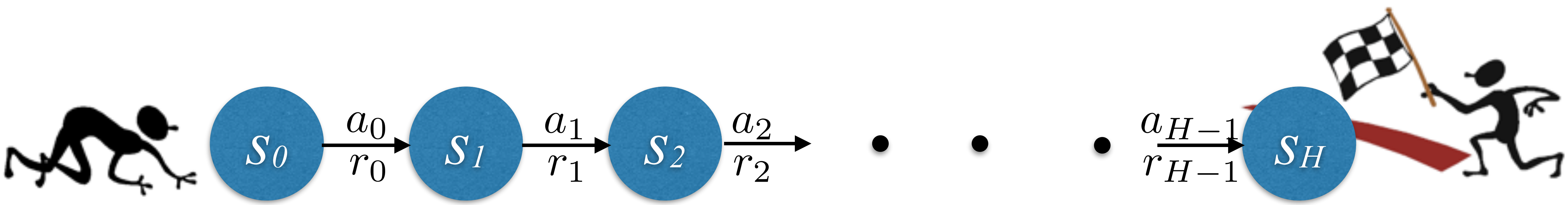

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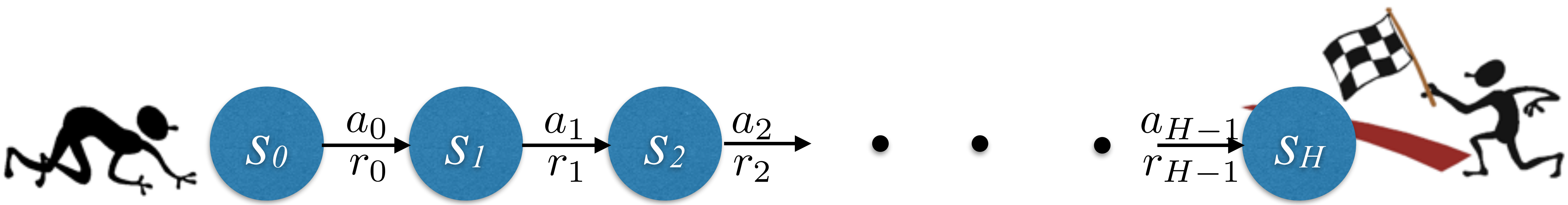

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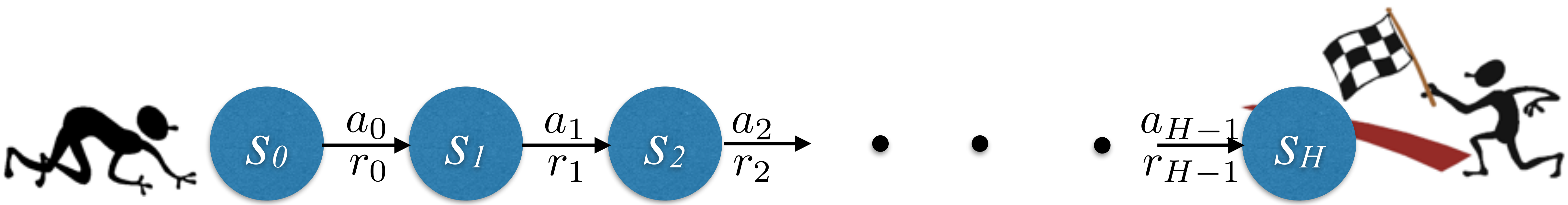

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- Regret

$$\text{Regret}(L) = \sum_{\ell=1}^{L} \left(V_0^*(s_0) - V^{\pi^\ell}(s_0) \right)$$

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open issue: how to explore efficiently alongside effective generalization methods

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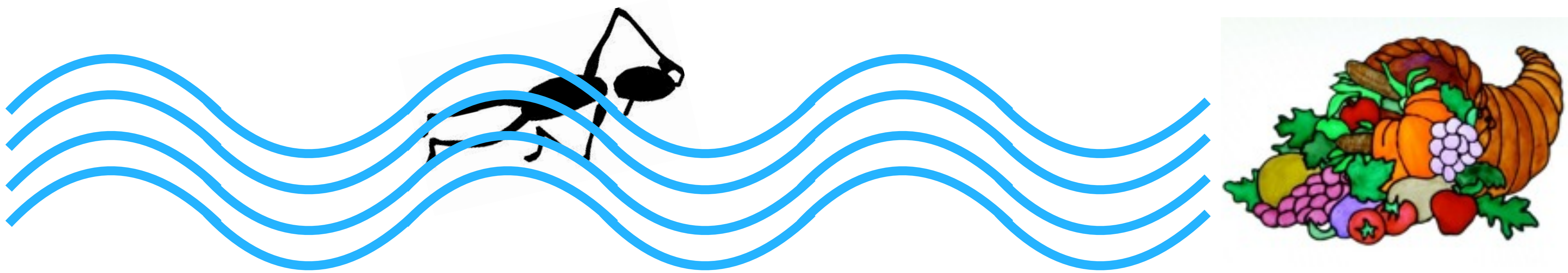

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myopia

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UCRL2

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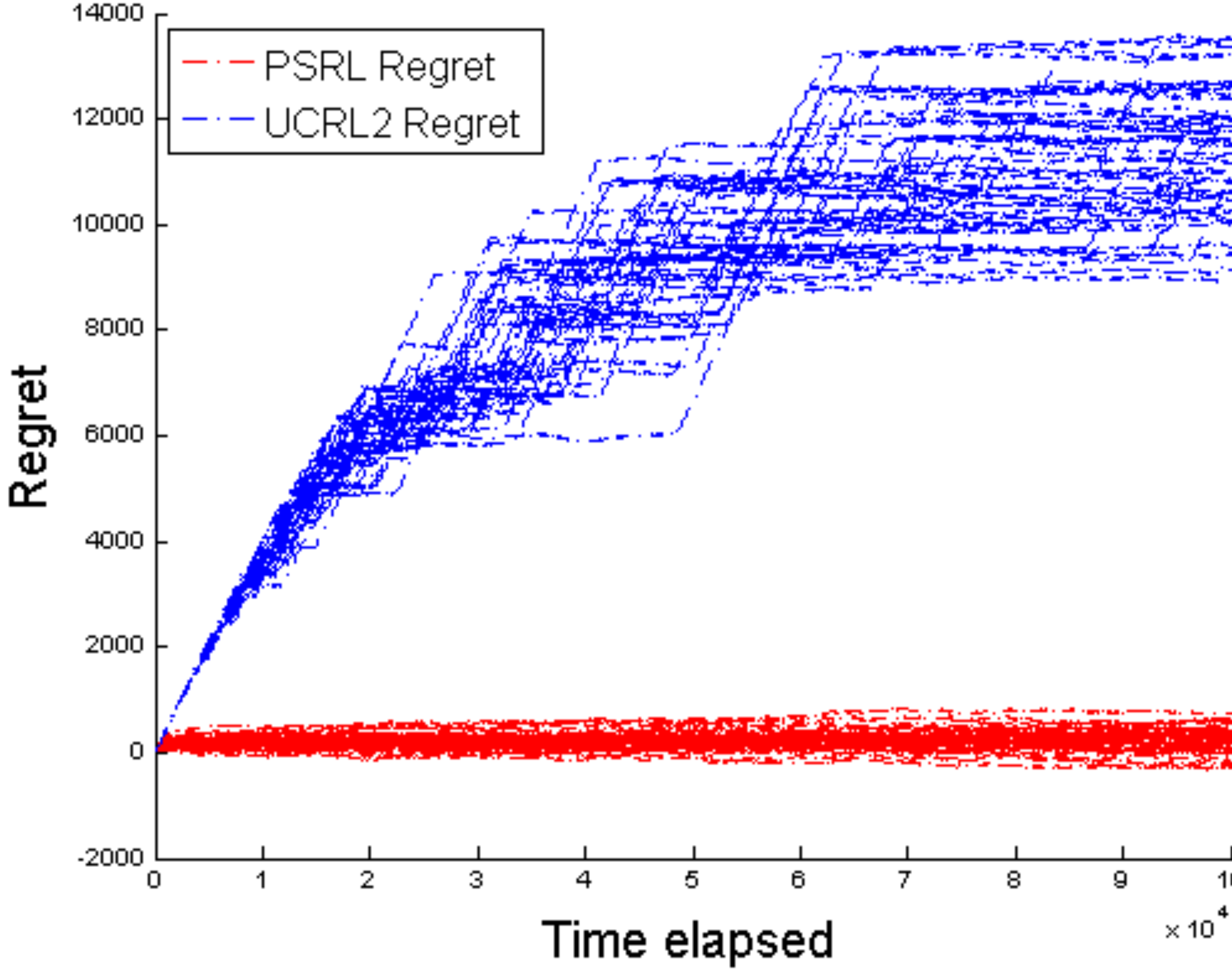
- "Near-optimal reinforcement learning"

UCRL2 versus PSRL

[Osband-Russo-VanRoy, 2014]



Why does UCRL2 perform so poorly?

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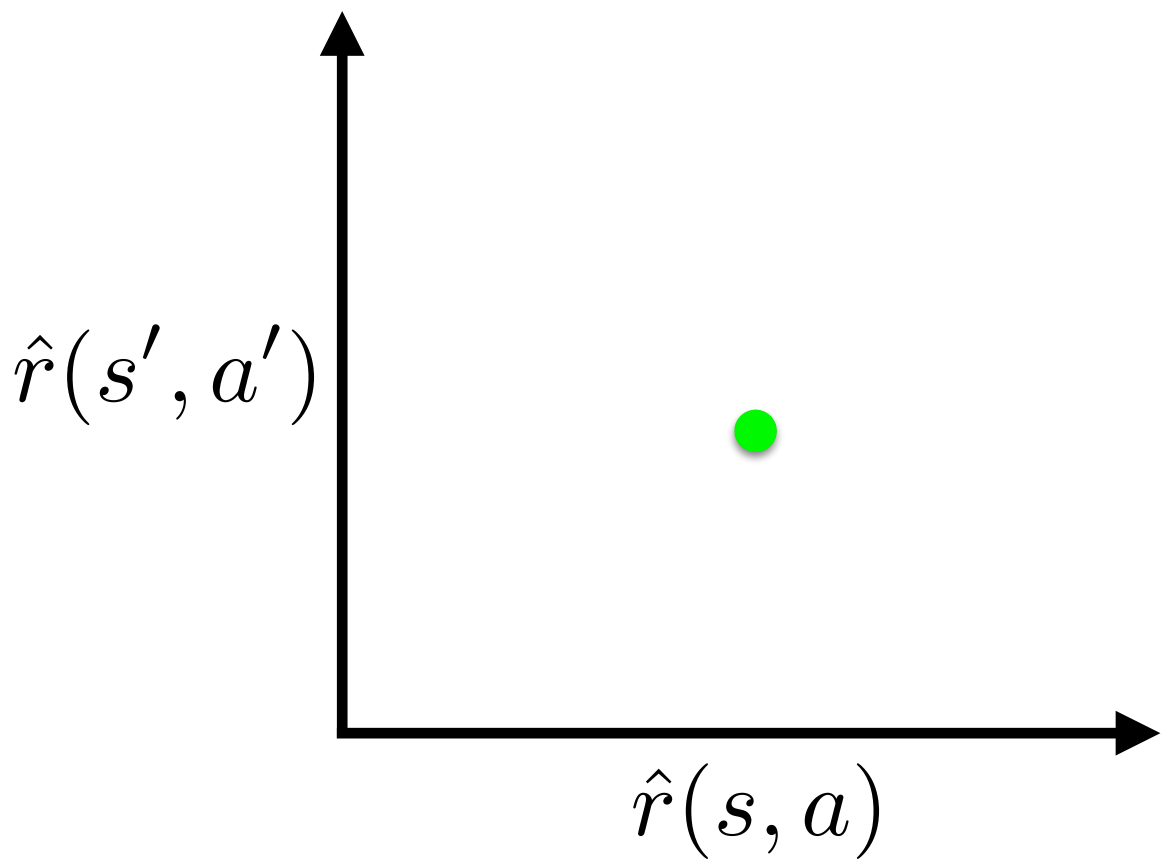

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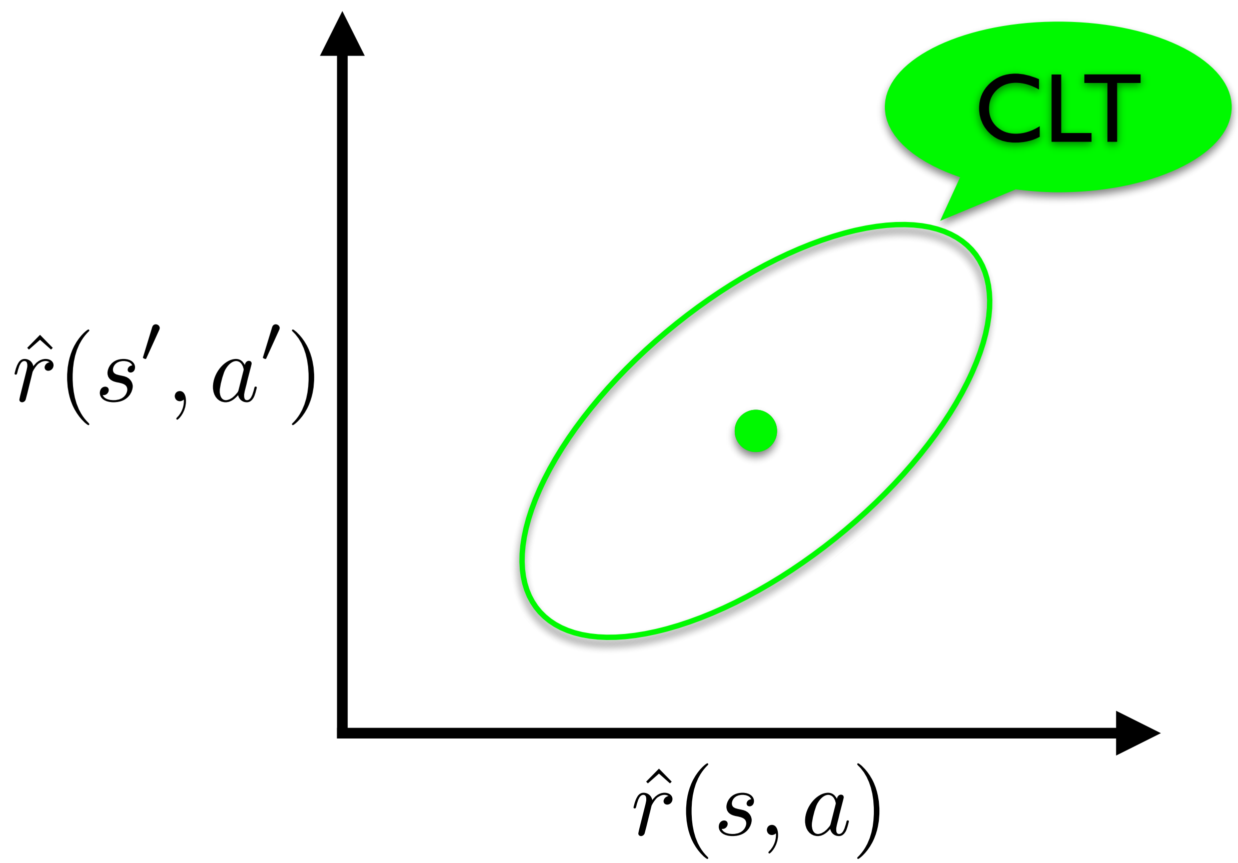

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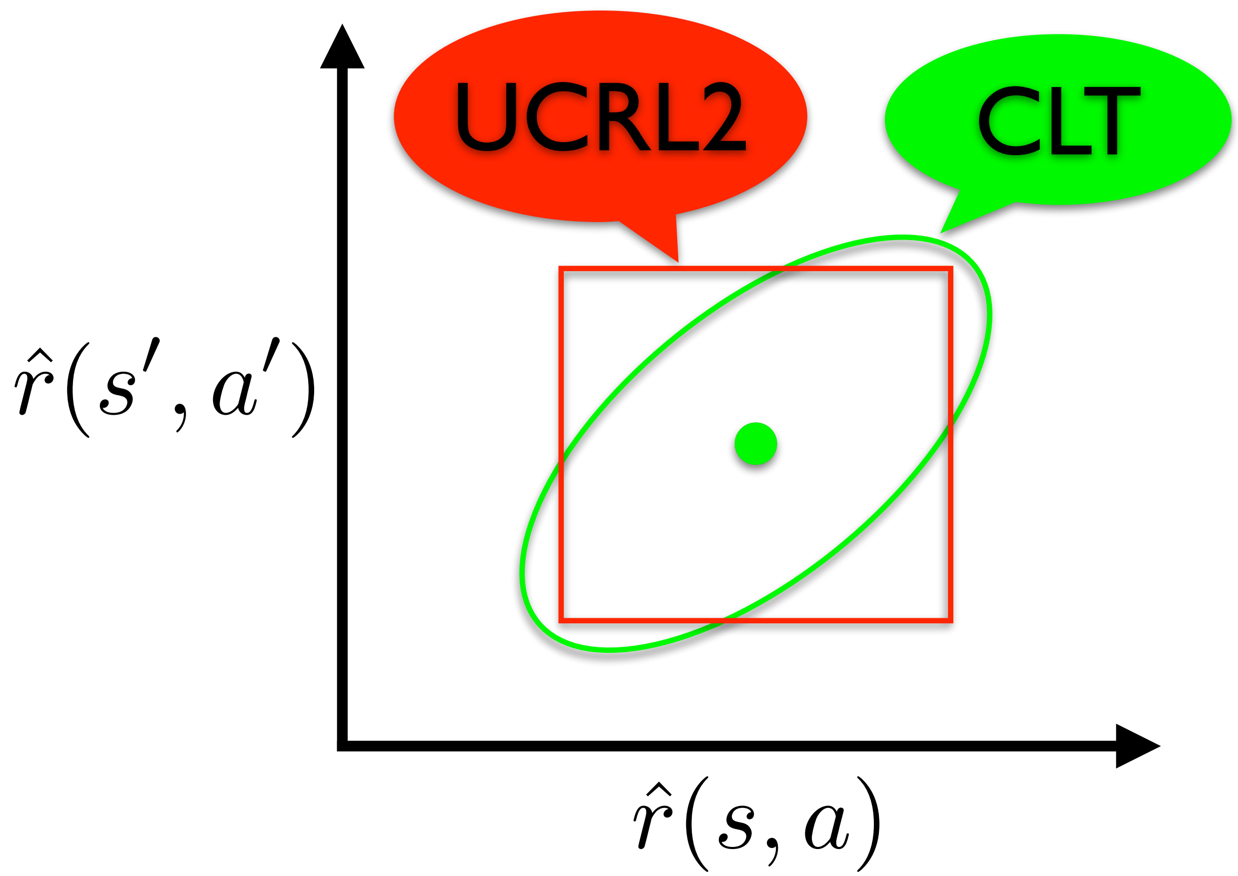

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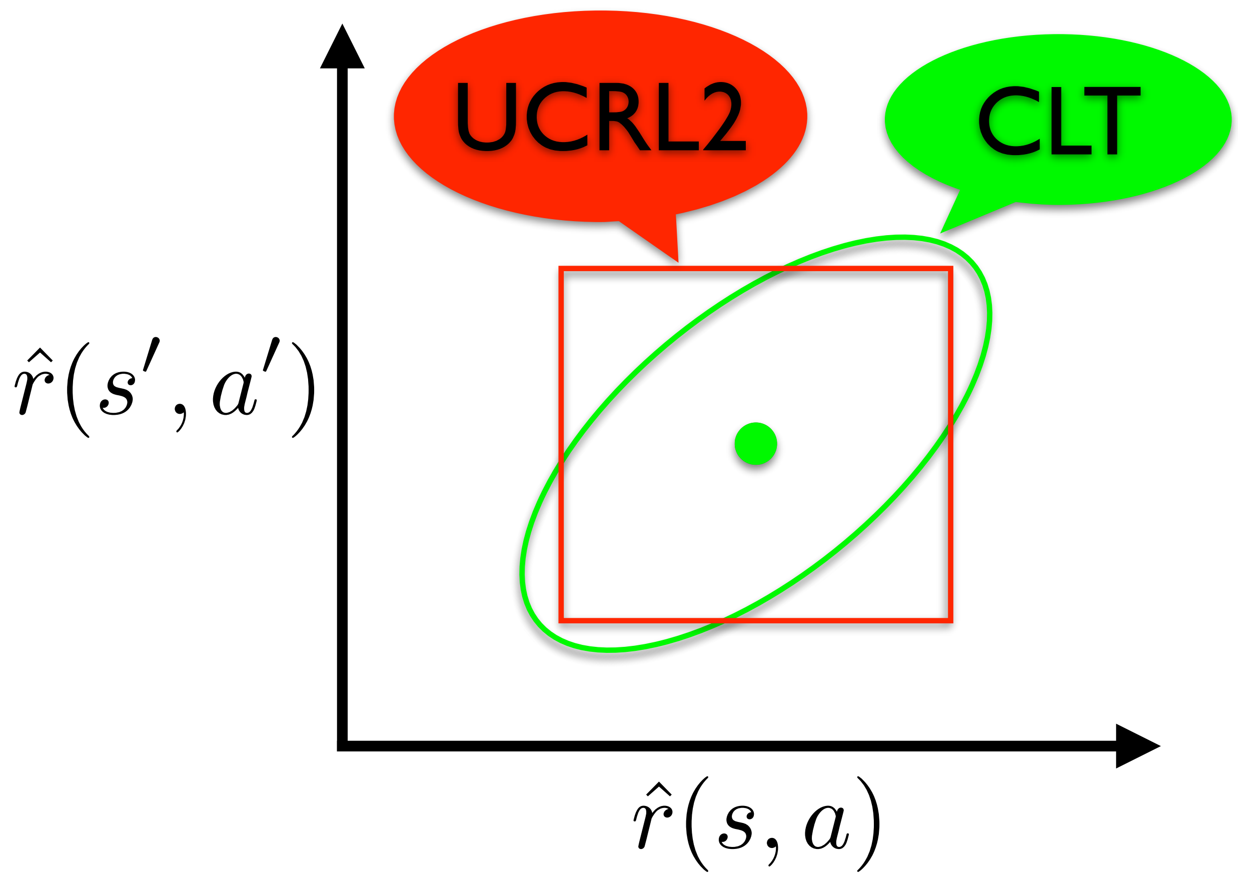

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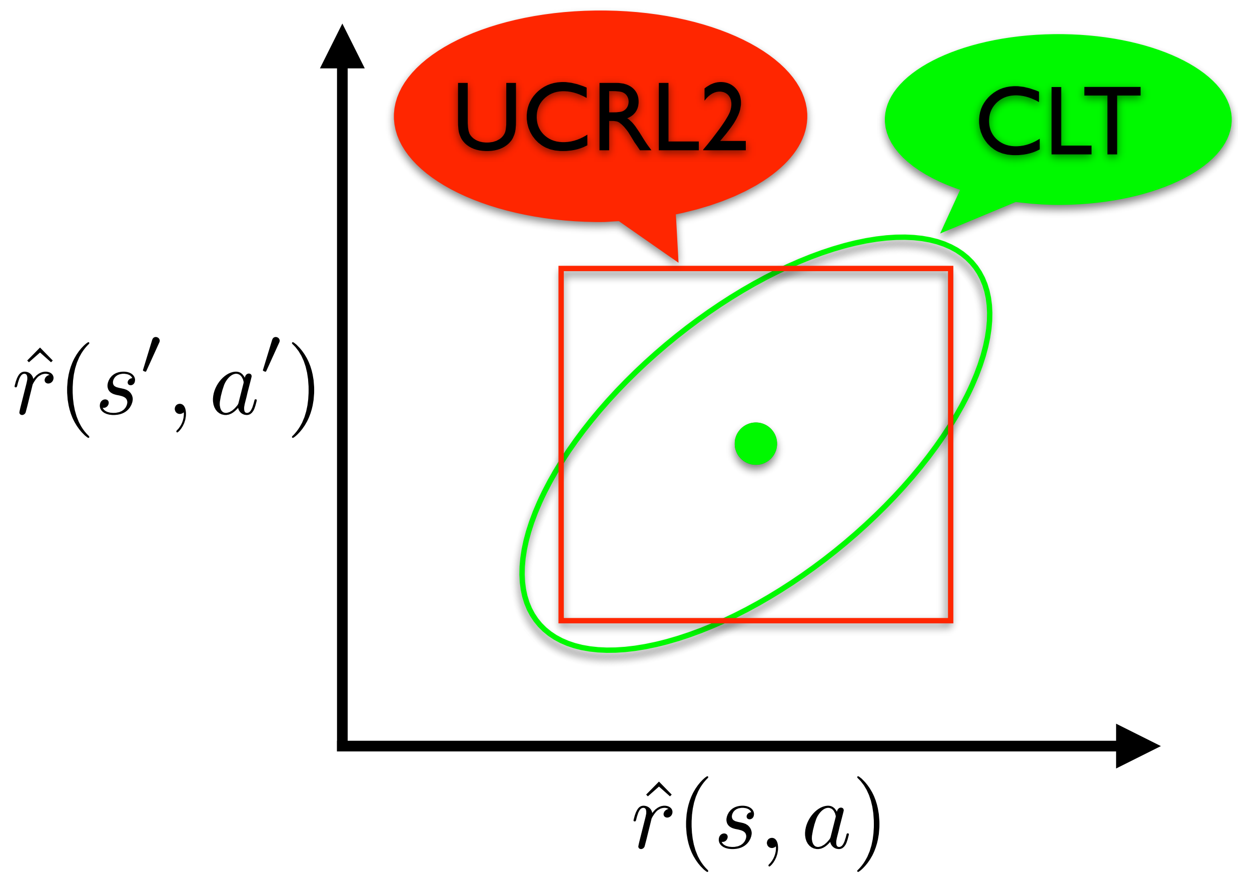

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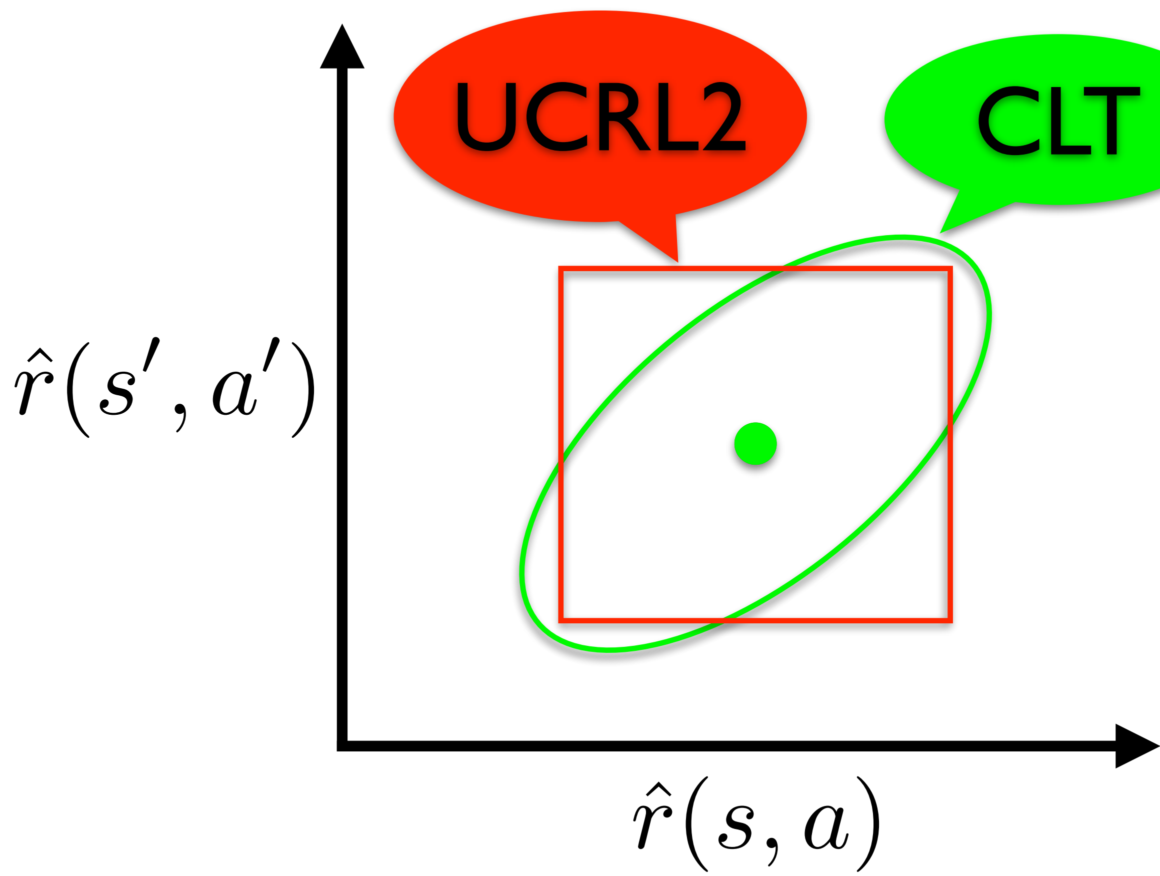

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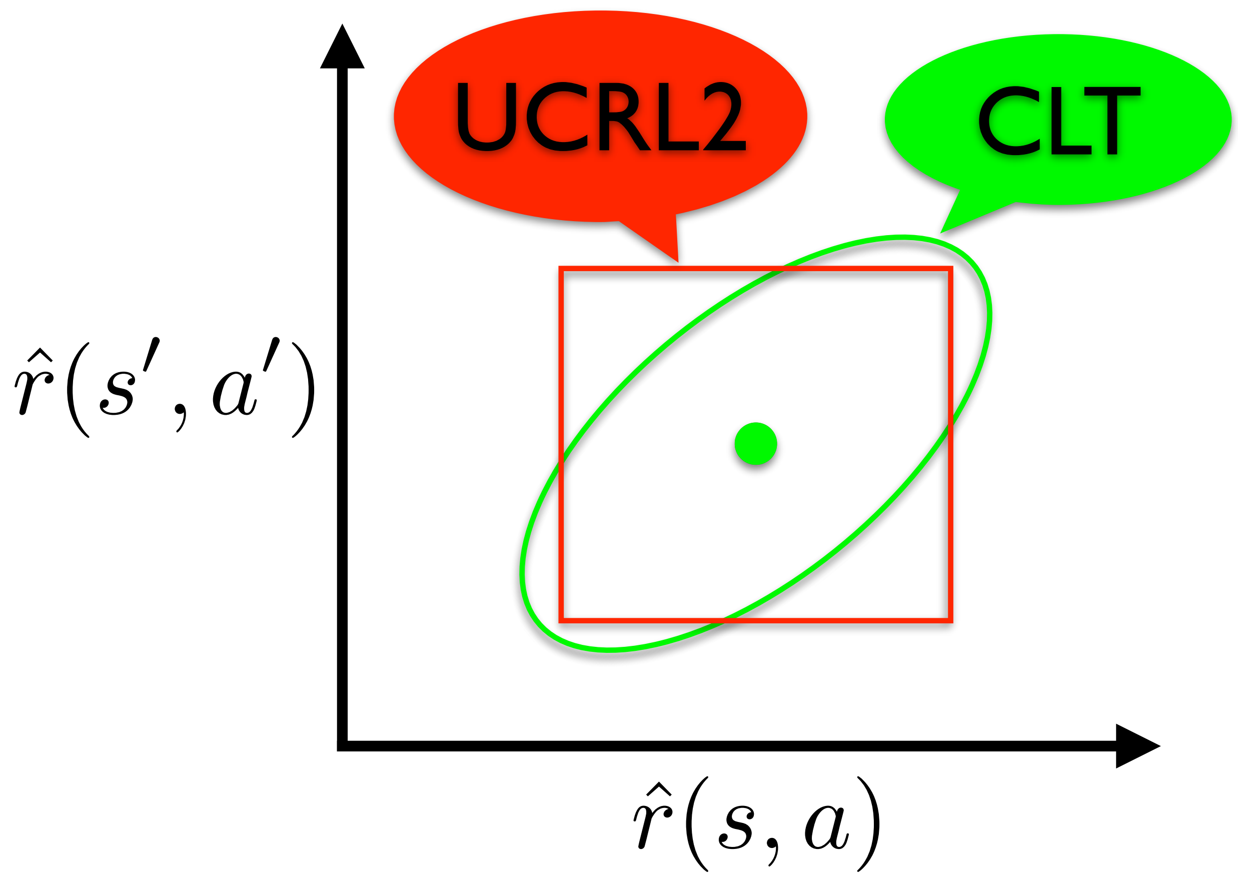

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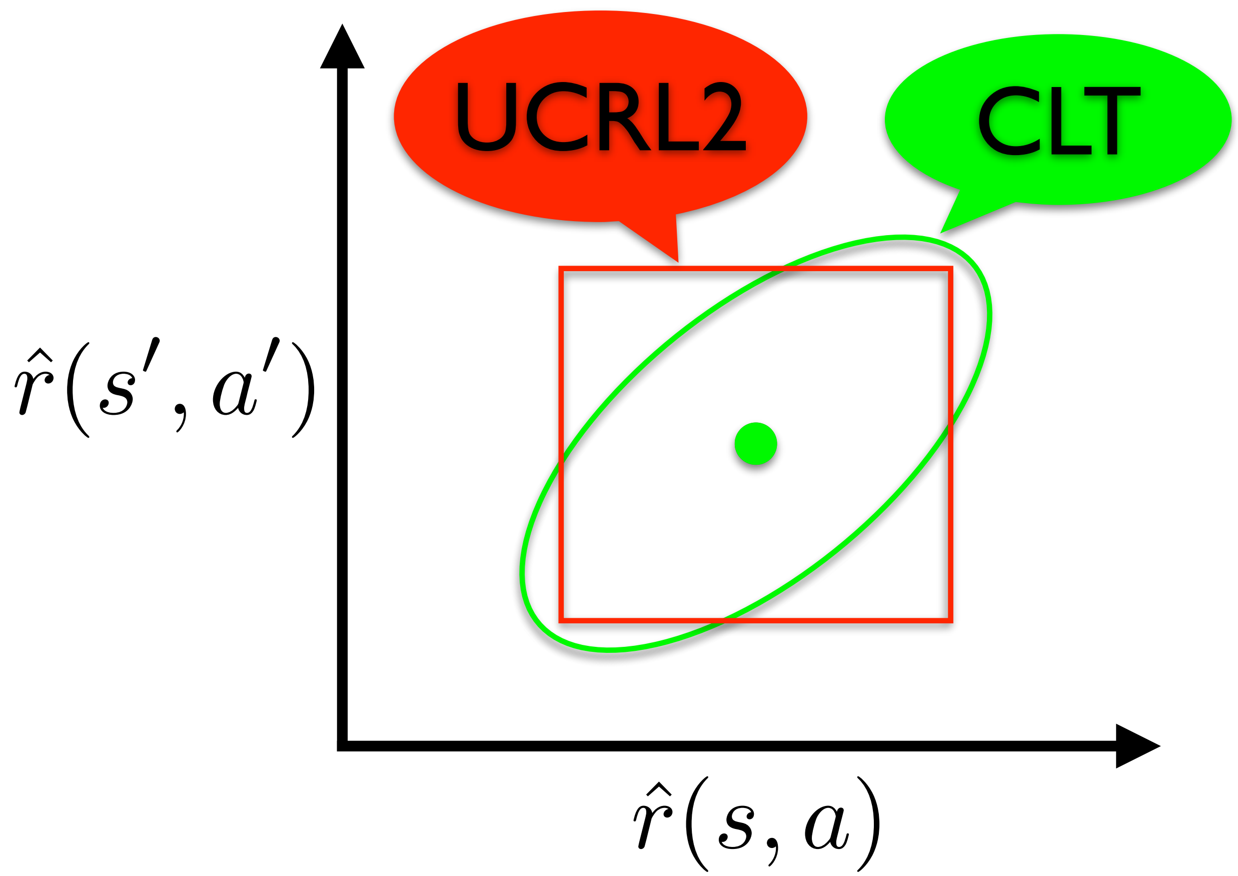

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  - PSRL approximates this

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[Strens, 2000]

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- Regret bound [Osband-Russo-VanRoy, 2014]

$$\mathbb{E}\left[\text{Regret}(L)\right] = \tilde{O}\left(HS\sqrt{AHL}\right)$$

How does PSRL plan to learn?

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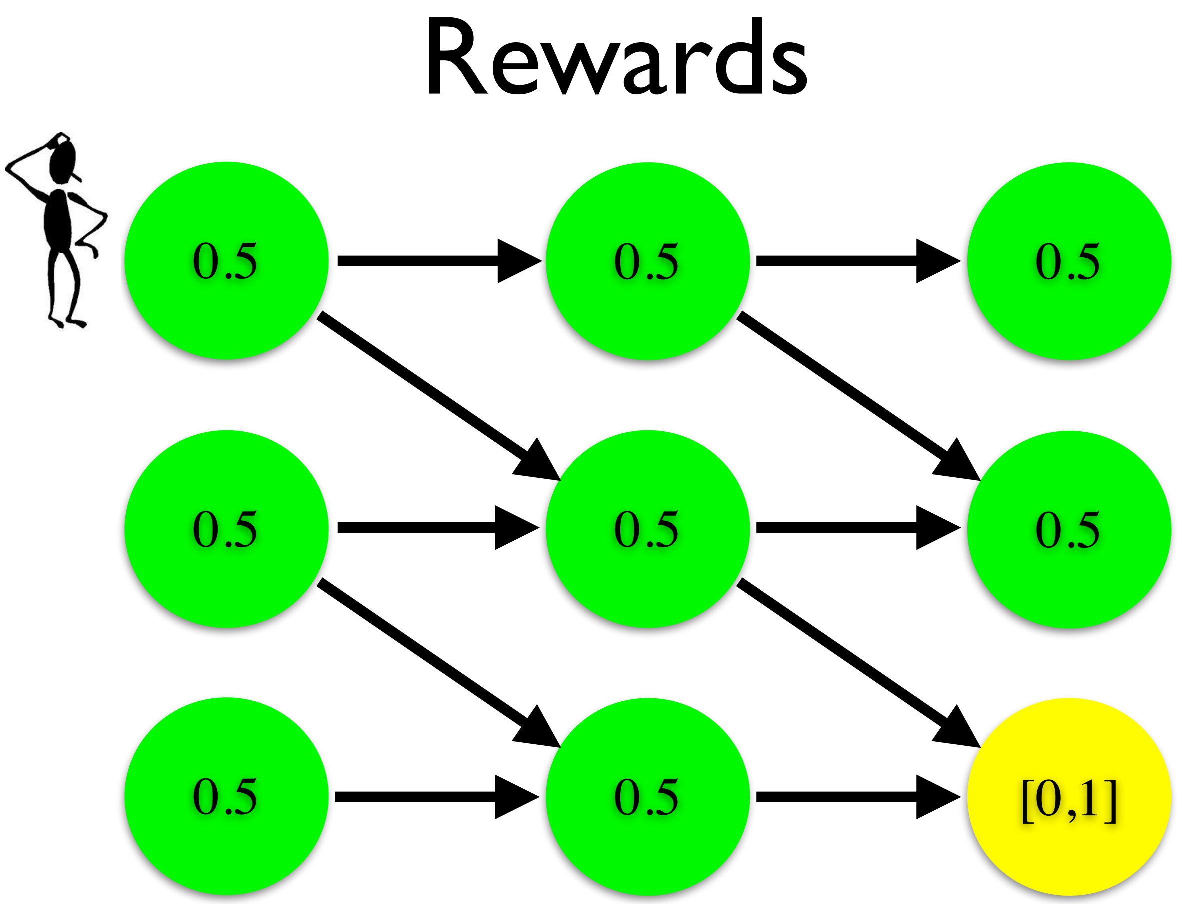

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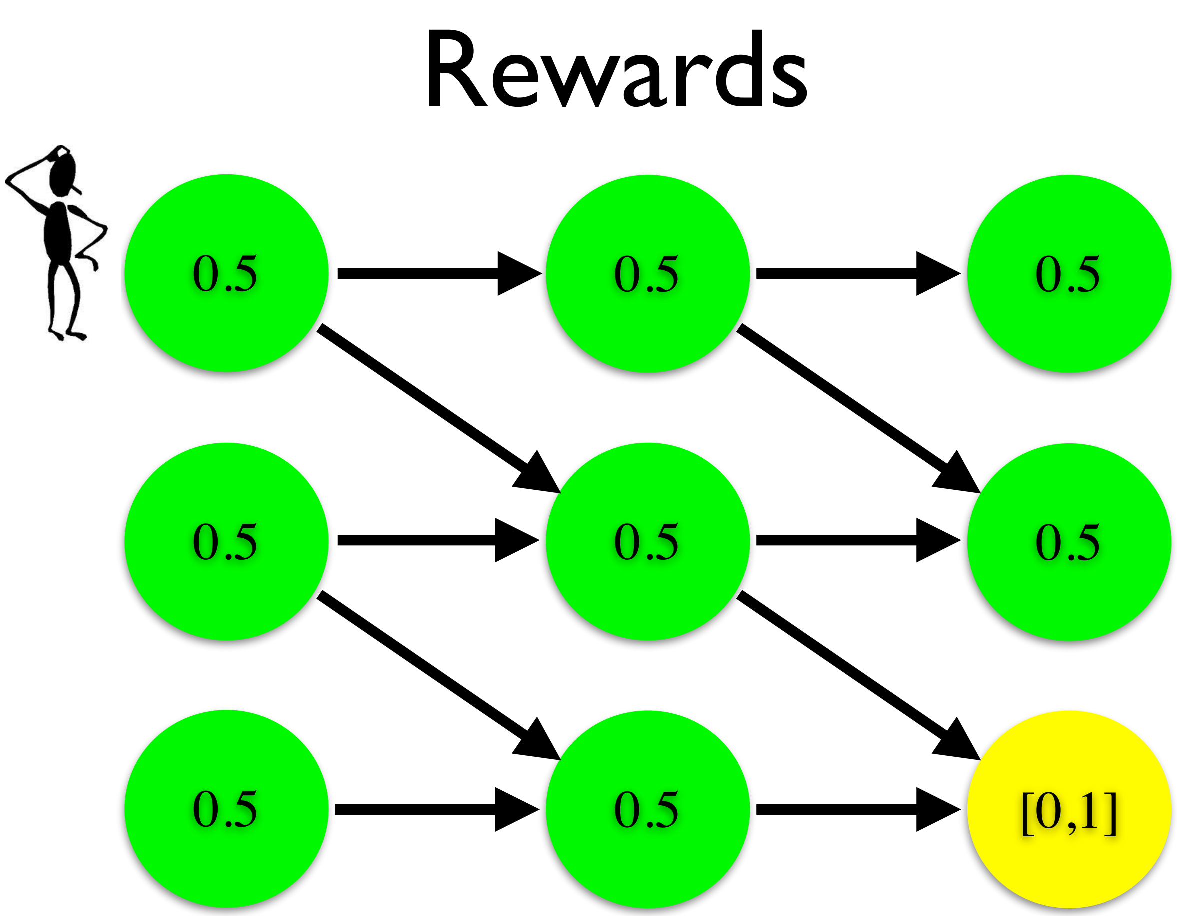

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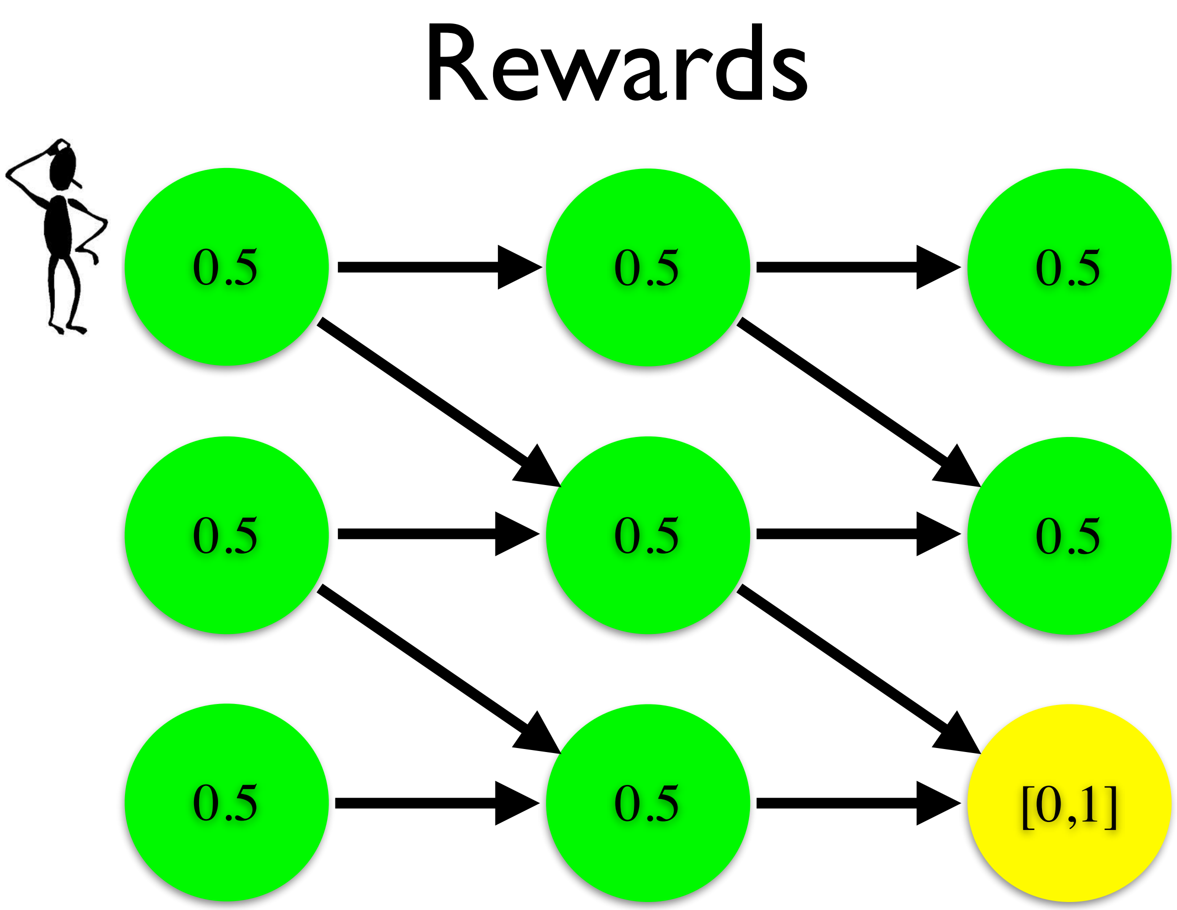

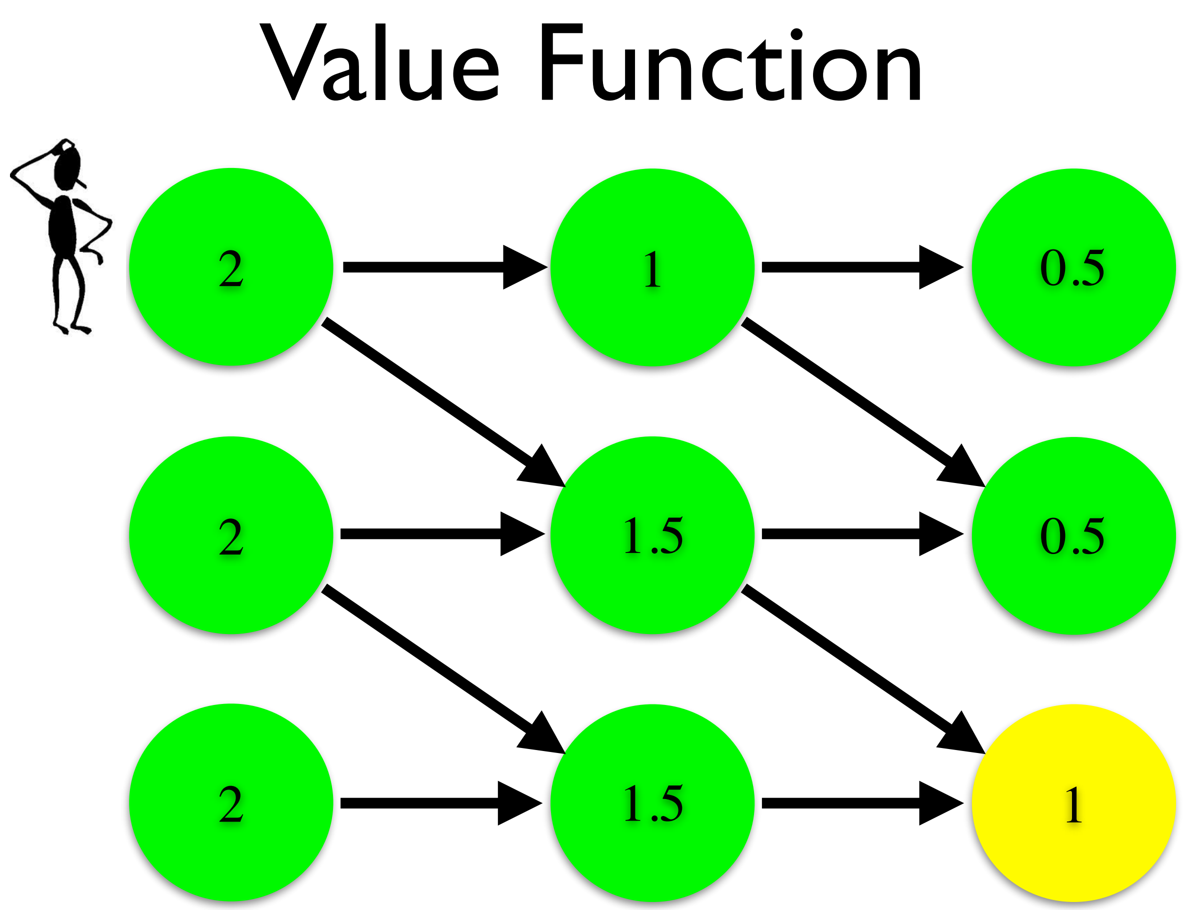

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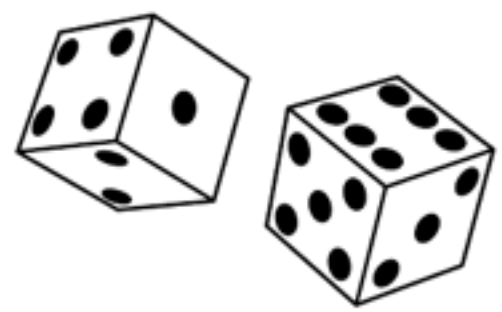
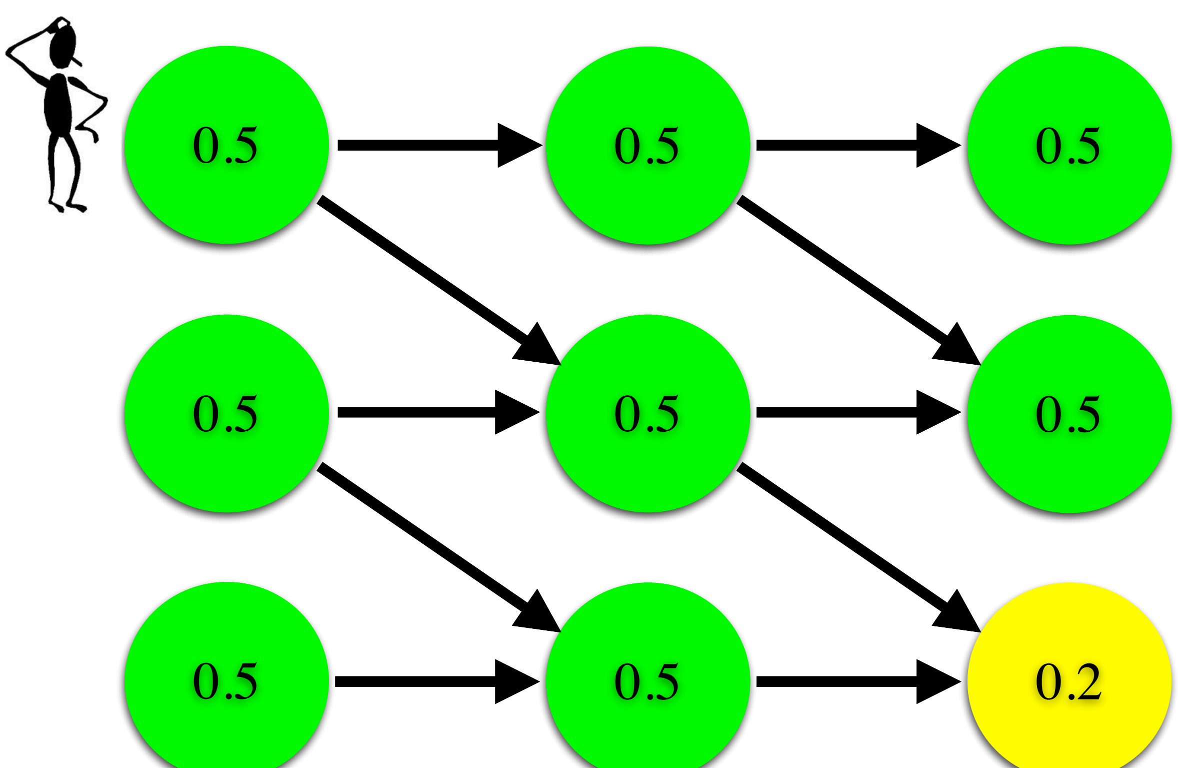
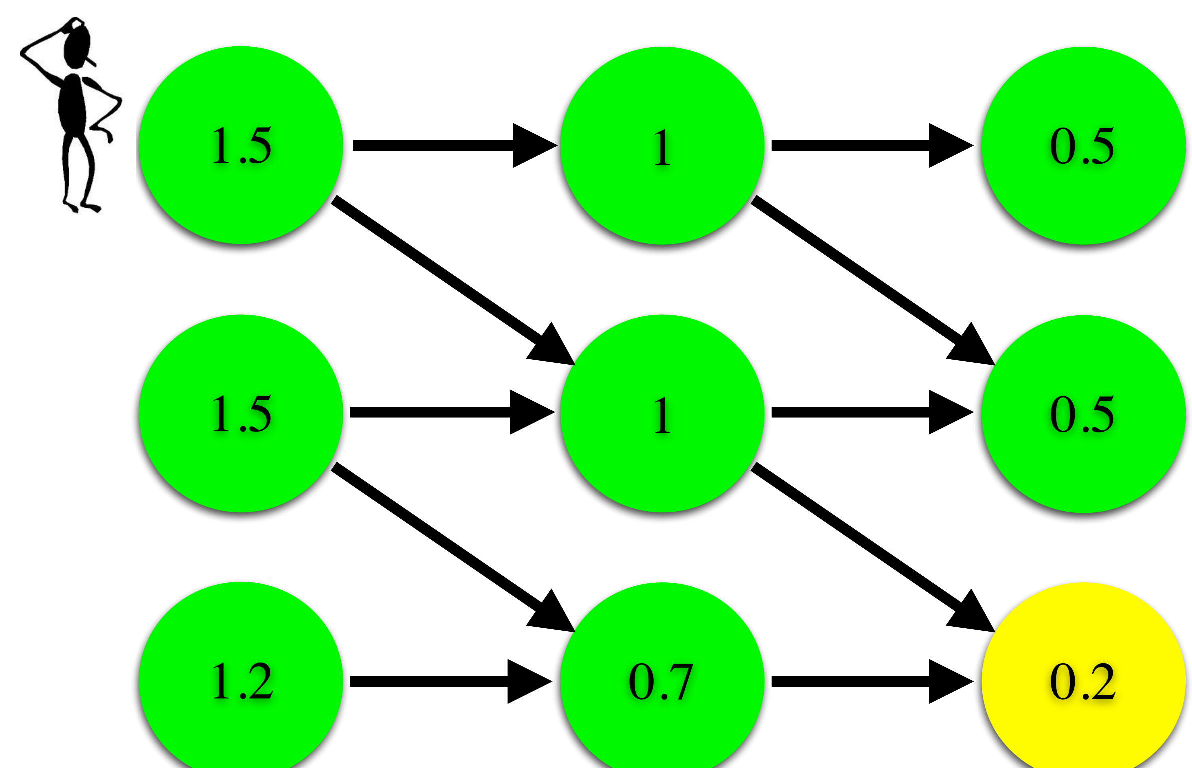


Rewards



Value Function



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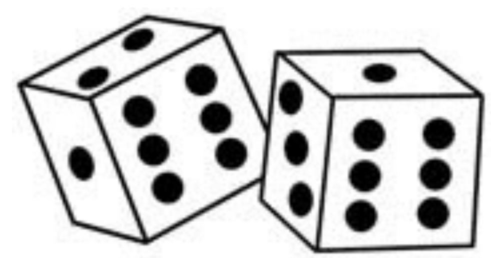

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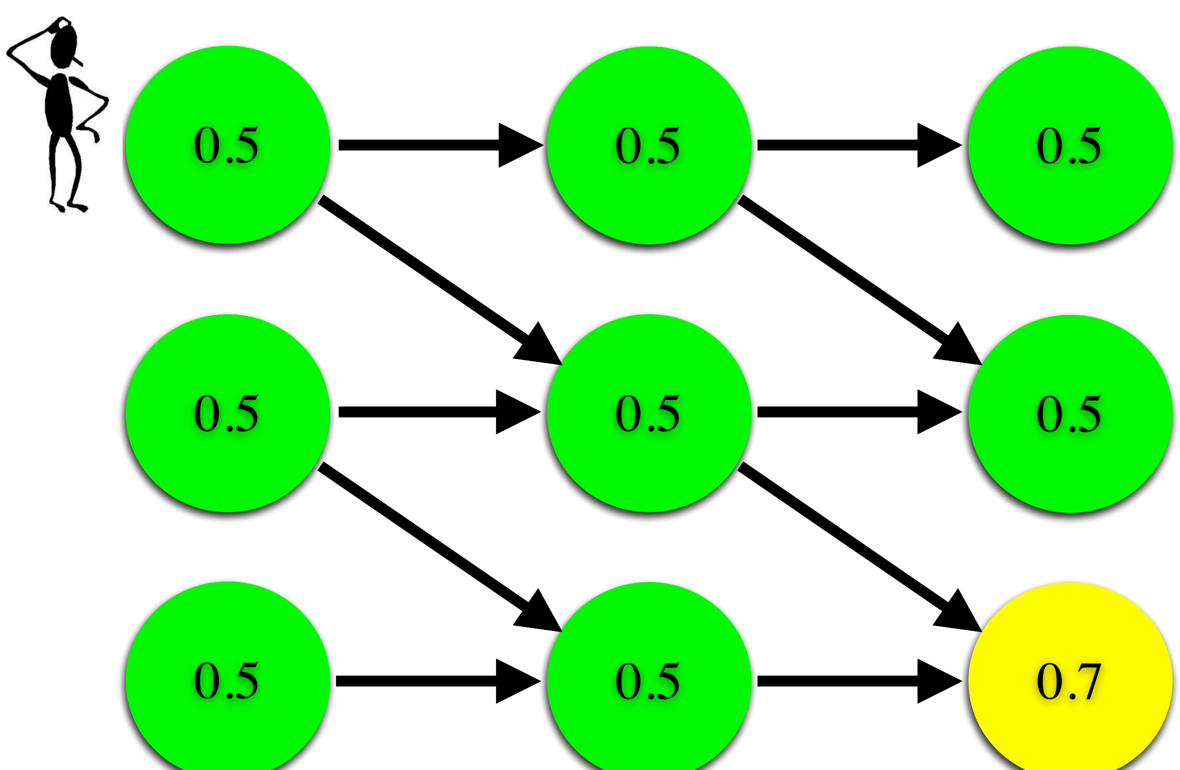

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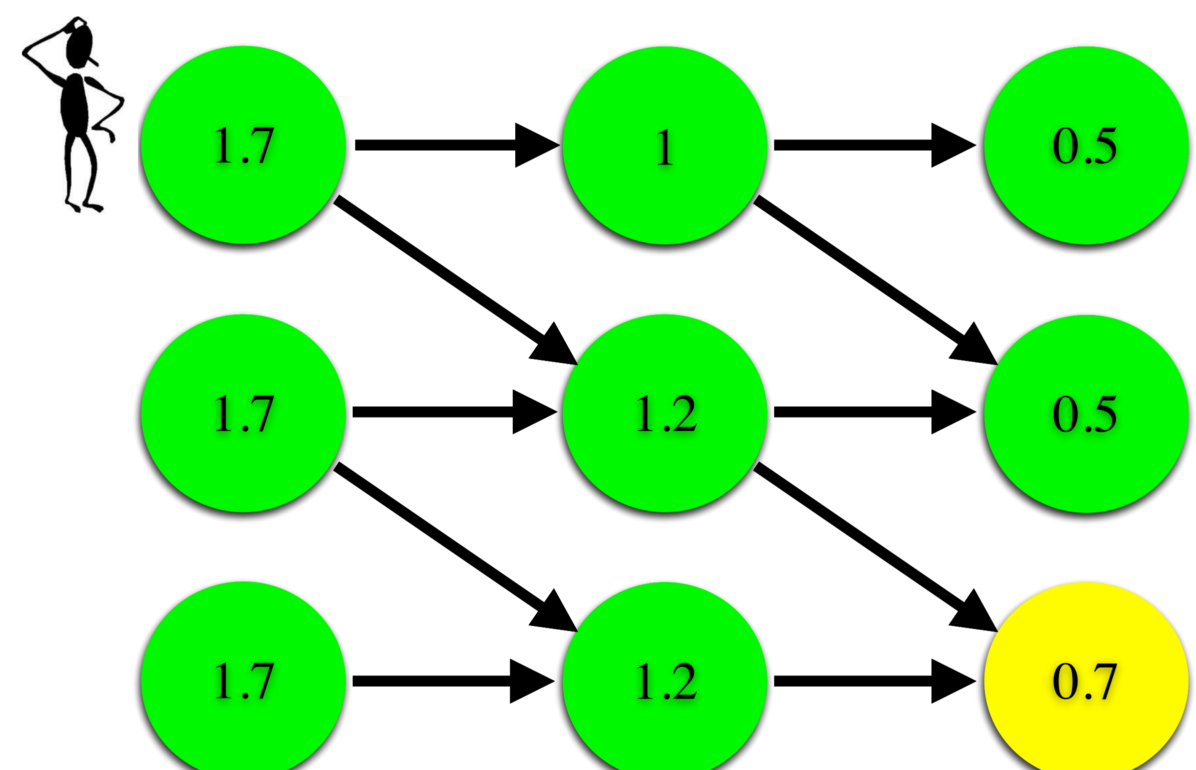

Beyond *Tabula Rasa* RL

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- Need to generalize
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- Need to generalize
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  - Parameterized value functions
- Simple case: linear combination of features

$$\tilde{Q}_h^{\theta_h}(s, a) = \sum_{k=1}^{K} \theta_{hk} \phi_{hk}(s, a)$$

Value Function Randomization

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- How to sample?
  - **Randomized least-squares value iteration**

RLSVI

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- Consider least-squares value iteration

$$\min_{\hat{\theta}_h} \left(\frac{1}{\sigma^2} \sum_{\ell=1}^{L} \left(\tilde{Q}_h^{\hat{\theta}_h}(s_h^\ell, a_h^\ell) - \left(r_h^\ell + \max_\alpha \tilde{Q}_{h+1}^{\hat{\theta}_{h+1}}(s_{h+1}^\ell, \alpha) \right) \right)^2 + \lambda \|\hat{\theta}_h\|_2^2 \right)$$

RLSVI

- Consider least-squares value iteration

$$\hat{\theta}_h \leftarrow \frac{1}{\sigma^2}\left(\frac{1}{\sigma^2}A^\top A + \lambda I\right)^{-1} A^\top b$$

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$$\Sigma_h \leftarrow \left(\frac{1}{\sigma^2}A^\top A + \lambda I\right)^{-1}$$

RLSVI

- Consider least-squares value iteration

$$\hat{\theta}_h \leftarrow \frac{1}{\sigma^2}\left(\frac{1}{\sigma^2}A^\top A + \lambda I\right)^{-1} A^\top b$$

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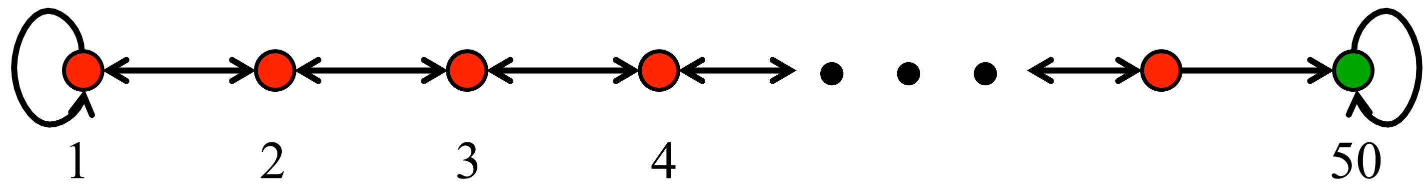
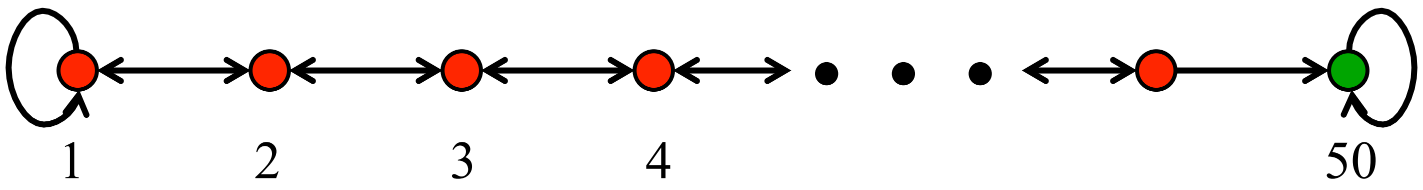
$$\hat{\theta}_h \sim \mathrm{N}(\overline{\theta}_h, \Sigma_h)$$

A Simple Example

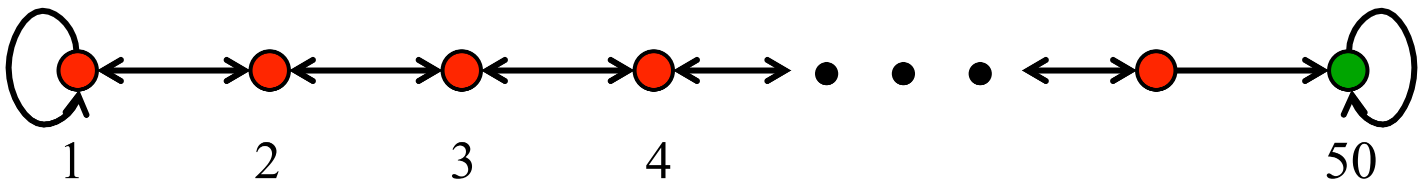


A Simple Example



- Deterministic MDP
  - Start at state 1
  - Actions: left or right
  - Horizon = 50 periods
  - Receive reward 1 only if at state 50

A Simple Example



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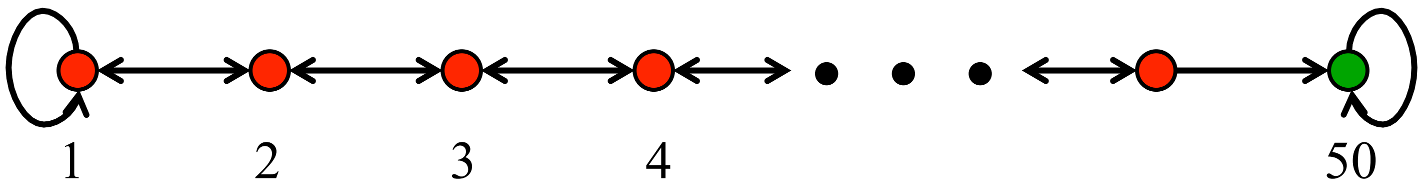
  - Horizon = 50 periods
  - Receive reward 1 only if at state 50

- What is the optimal strategy?

A Simple Example



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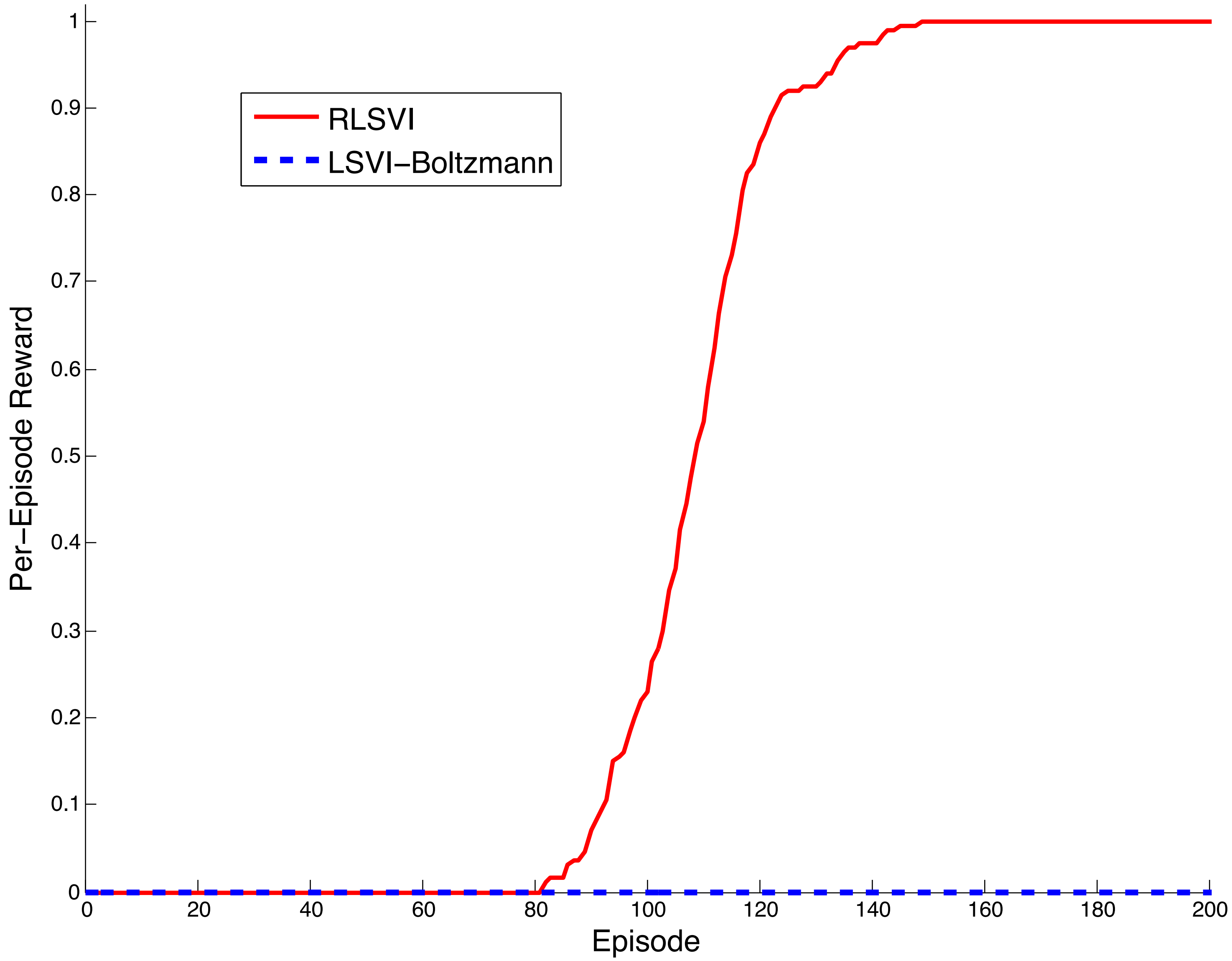
  - Actions: left or right
  - Horizon = 50 periods
  - Receive reward 1 only if at state 50

- What is the optimal strategy?

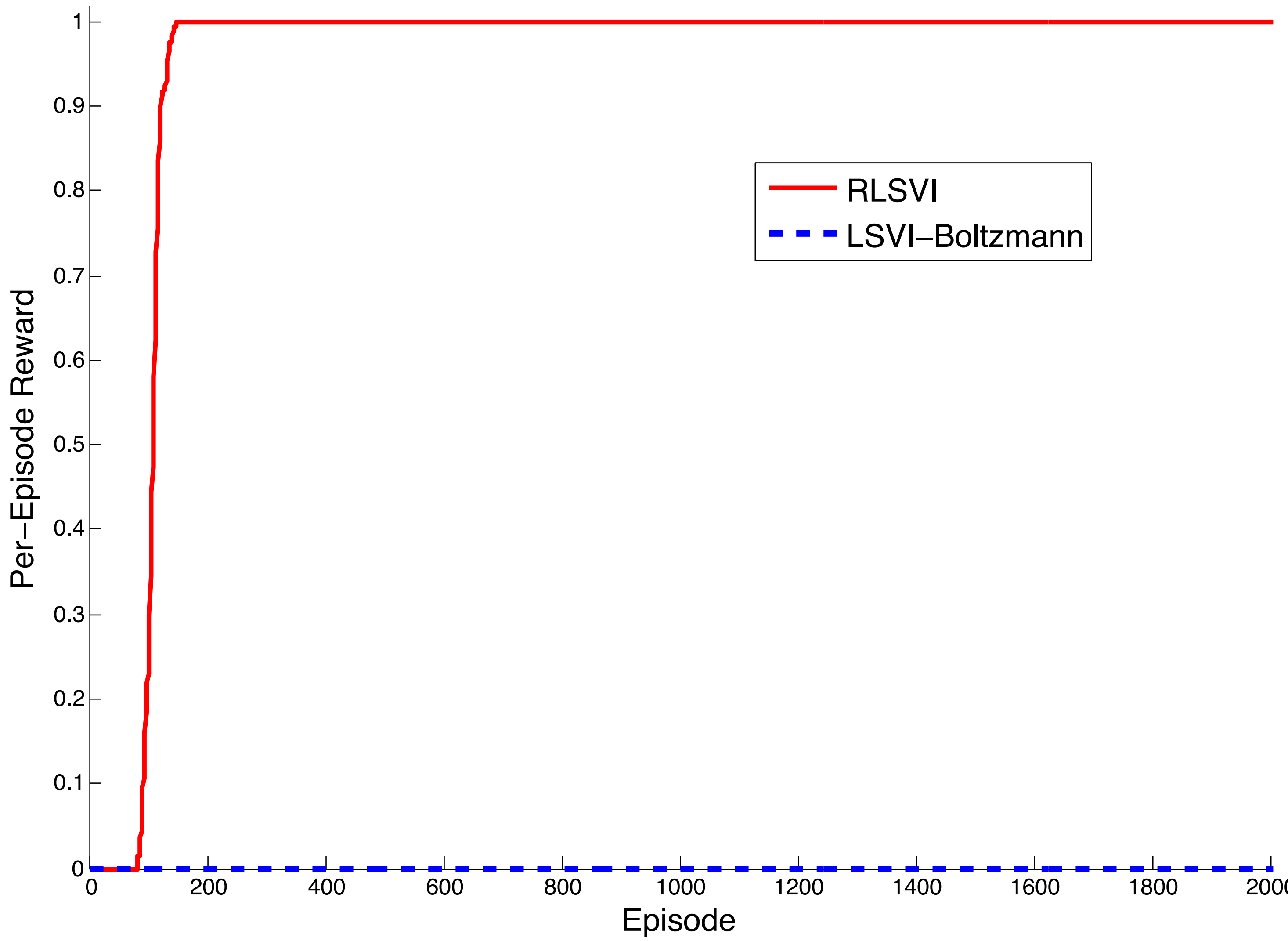
- $\tilde{Q}_h^{\theta_h}$ spans a random affine subspace that contains Q_h^* and constant functions

Boltzmann versus VF Randomization



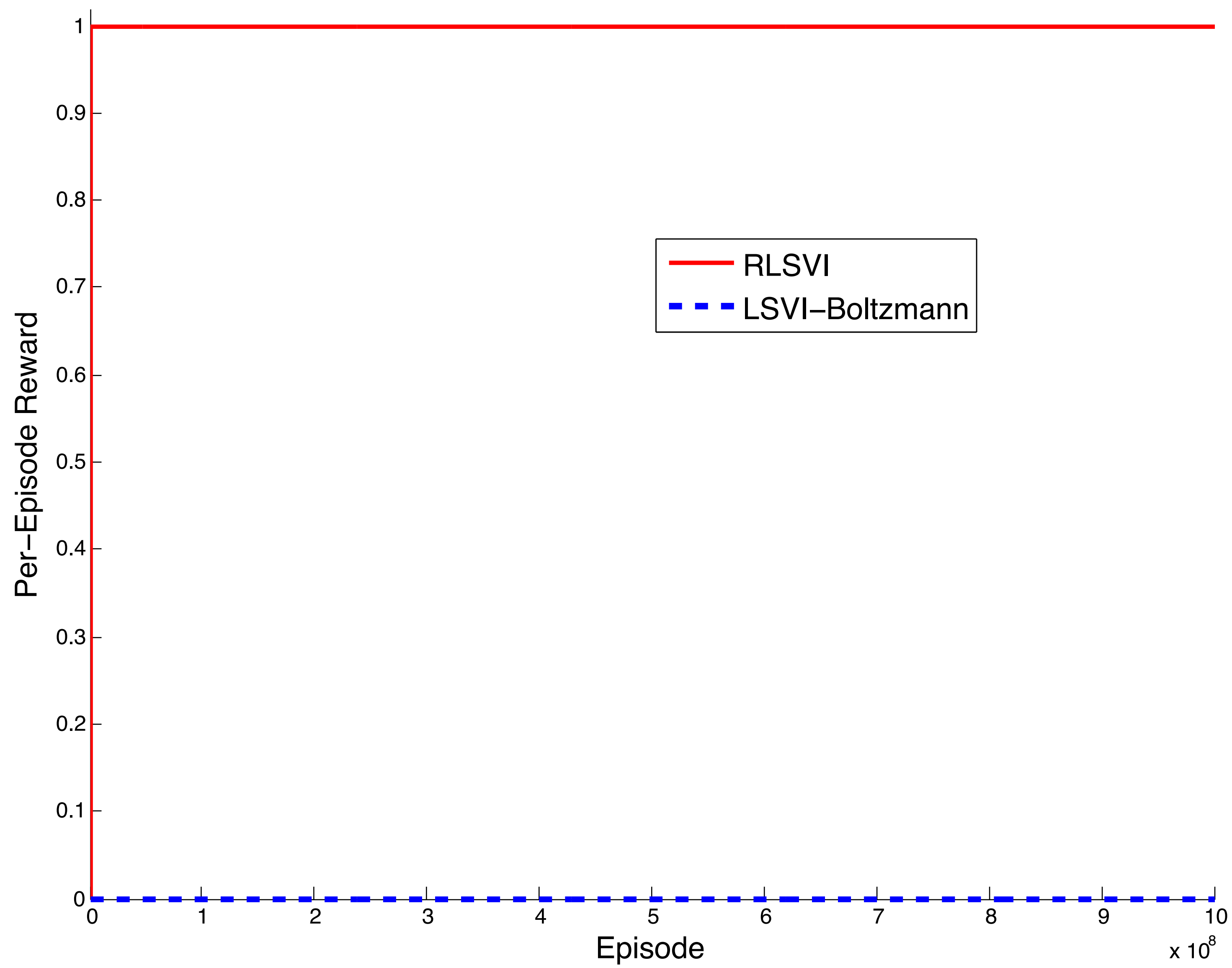
$K = 10$ features

Boltzmann versus VF Randomization



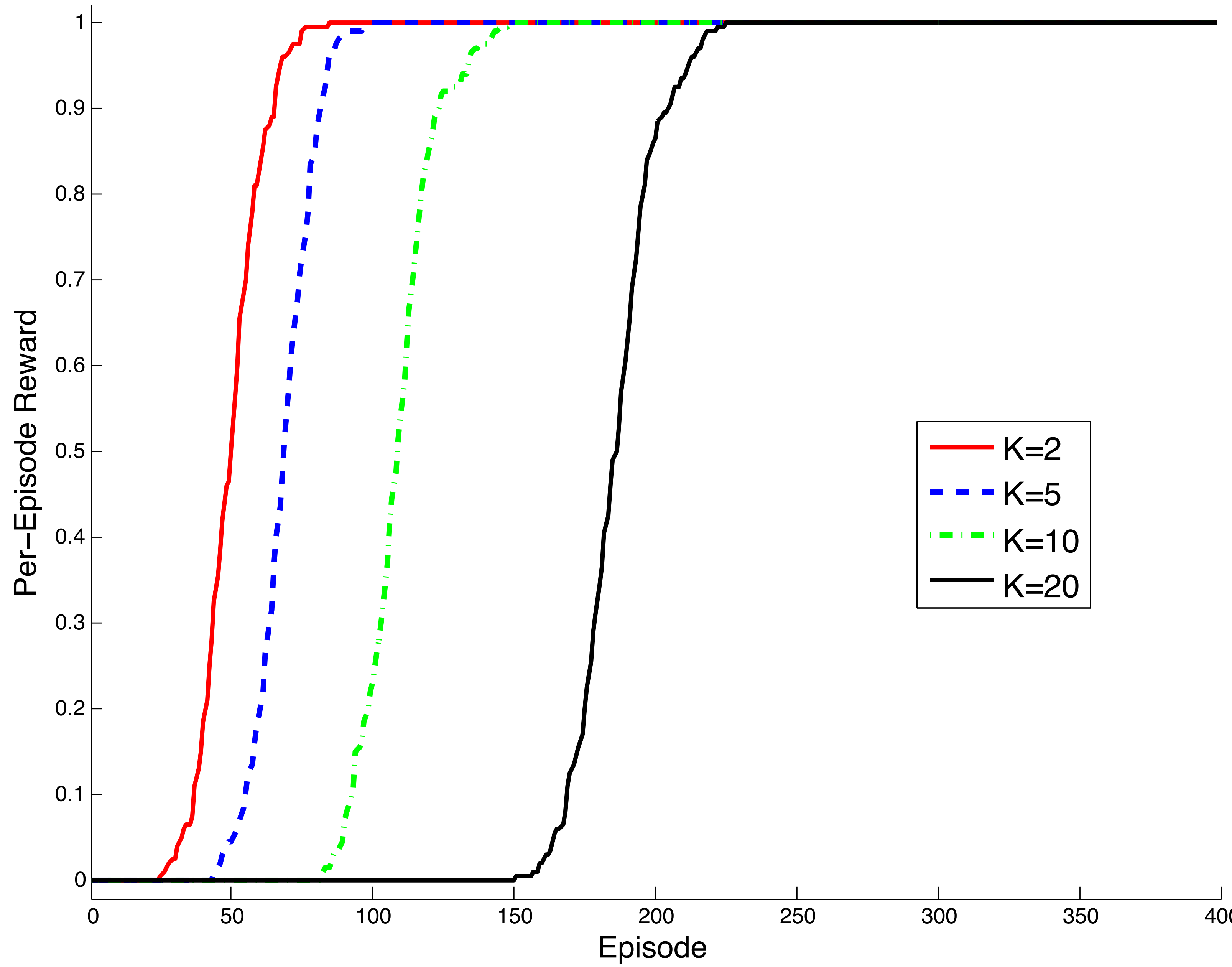
$K = 10$ features

Boltzmann versus VF Randomization

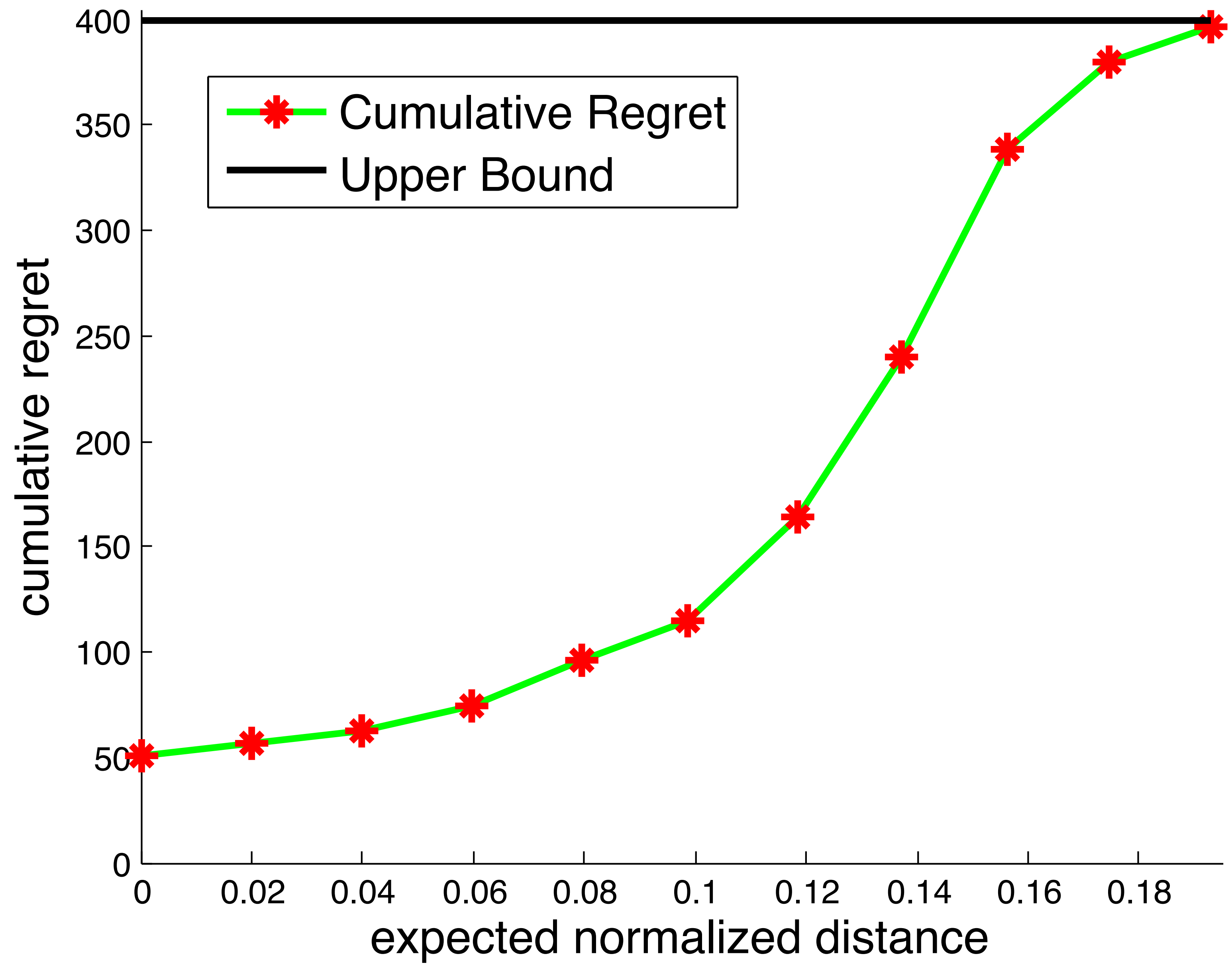


$K = 10$ features

Varying the Number of Features



Agnostic Learning



$K = 11$ features

Understanding RLSVI: *Tabula Rasa* Case

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$$\text{RLSVI} \approx \text{PSRL}$$

Understanding RLSVI: *Tabula Rasa* Case

RLSVI $\approx$ PSRL

- Assume unknown deterministic rewards $r_h(x, a)$

Understanding RLSVI: *Tabula Rasa* Case

RLSVI $\approx$ PSRL

- Assume unknown deterministic rewards $r_h(x, a)$
- PSRL value computation

$$\hat{Q}_h(s, a) \leftarrow \tilde{r}_h(s, a) + \sum_{s'} \tilde{p}_{ss'}(a) \max_{\alpha} \hat{Q}_{h+1}(s', \alpha)$$

Understanding RLSVI: *Tabula Rasa* Case

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$$\hat{Q}_h(s,a) \leftarrow \overline{r}_h(s,a) + \sum_{s'} \overline{p}_{ss'}(a) \max_{\alpha} \hat{Q}_{h+1}(s', \alpha) + \text{noise}$$

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- RLSVI value computation with $\lambda = 0$

$$\hat{Q}_h(s, a) \leftarrow \frac{1}{n_h(s, a)} \sum_{\ell} \left(r_h^\ell + \max_{\alpha} \hat{Q}_{h+1}(s_{h+1}^\ell, \alpha) \right) + \text{noise}'$$

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- RLSVI value computation with $\lambda = 0$

$$\hat{Q}_h(s,a) \leftarrow \overline{r}'_h(s,a) + \sum_{s'} \overline{p}'_{ss'}(a) \max_{\alpha} \hat{Q}_{h+1}(s', \alpha) + \text{noise}'$$

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- For PSRL with uniform prior

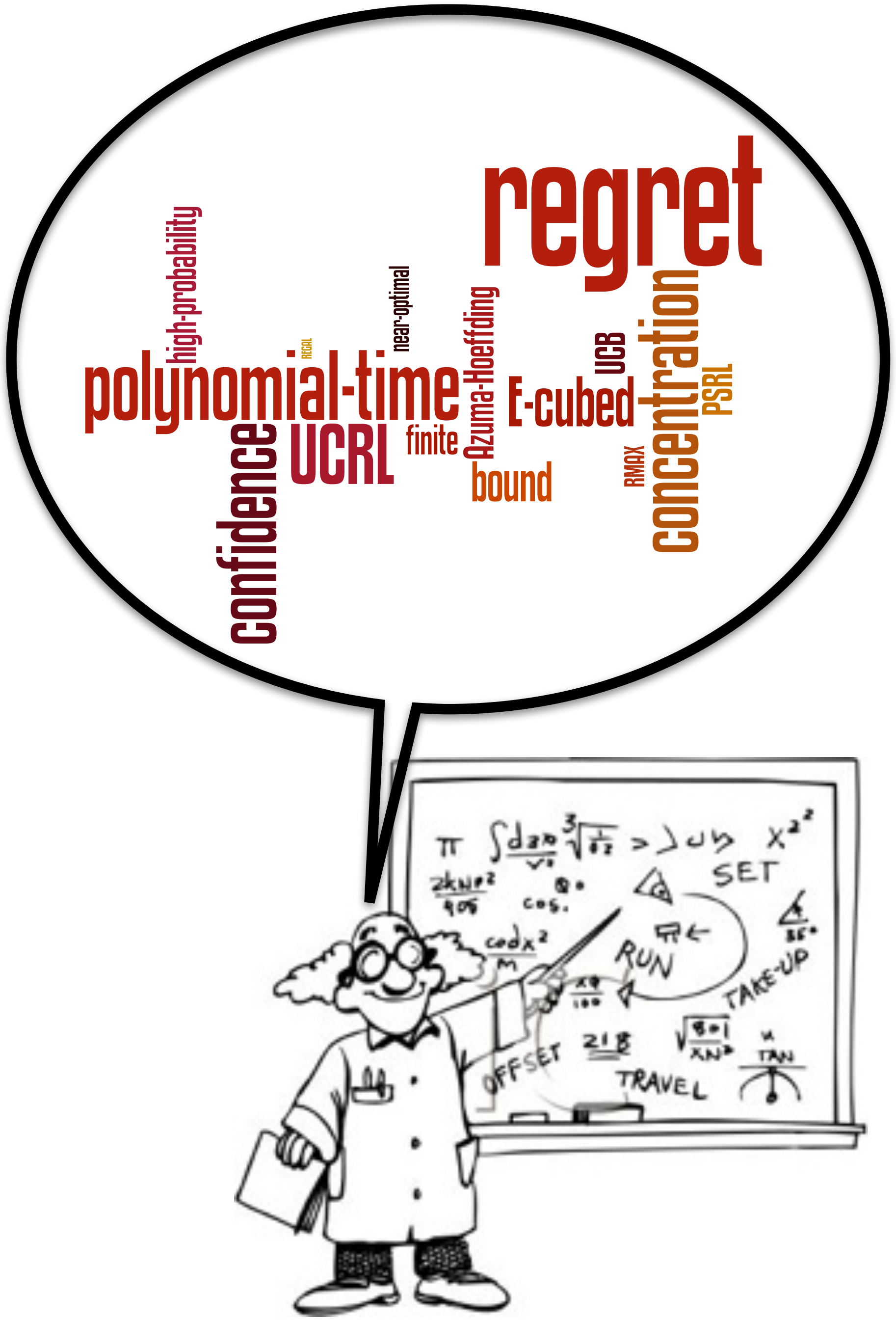
$$\overline{r}_h(s,a) = \overline{r}'_h(s,a)$$

$$\overline{p}_h(s,a) = \left(1 - \frac{1}{n_h(s,a)+1}\right)\overline{p}'_h(s,a) + \frac{1}{n_h(s,a)+1}\frac{1}{S} \rightarrow \overline{p}'_h(s,a)$$


SET
RUN
TAKE-UP
OFFSET
TRAVEL
TAN


SET
cos.
RUN
TAKE-UP
OFFSET
218
TRAVEL
TAN


COLA


regret
polynomial-time
confidence
UCRL
high-probability
near-optimal
Azuma-Hoeffding
finite
bound
E-cubed
UCB
RMAX
concentration
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SARSA
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LSPI
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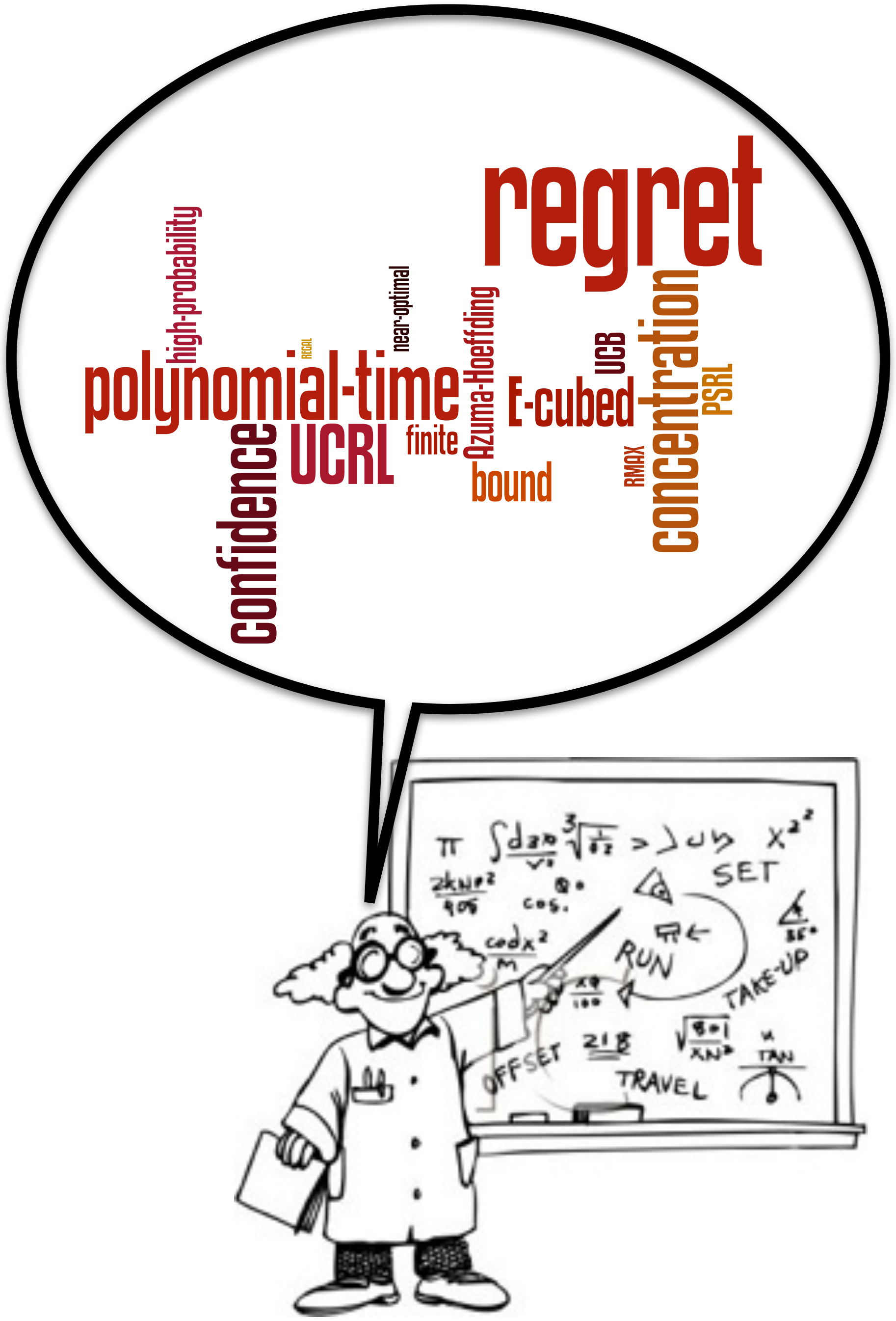
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