Majority Judgement A New Voting Method

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(Joint work with Michel Balinski) LNMB conferences The mathematics of operations research January 14-16, 2014

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- Incompatibility Between Electing and Ranking
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- Theory of Majority Judgement
- Experimental evidences

Conclusions

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Majority principle?

• Every democratic country and institution pretends to elect its representatives by a majority principle, though the rules by which it does so vary.

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- The methods of voting that are used differ in two ways : (1) how voters express their opinions the inputs and (2) how the various opinions are amalgamated the outputs .

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- The methods of voting that are used differ in two ways : (1) how voters express their opinions the inputs and (2) how the various opinions are amalgamated the outputs .
- Every one of the methods is meant to be and is commonly referred to being a "majority decision."

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A challenge

The legitimating force of the majority rule is so pervasive that we often do not notice it and rarely do we question it : We usually take it for granted. [...] It is much too powerful to make it vulnerable to a philosophical challenge.

(Wojciech Sadurski, 2008).

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It is our contention to challenge the current philosophical view of what constitutes a majority decision and propose an alternative.

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A tentative definition

Majority decision should be the answer to a specific, operationally pertinent question with which more than half of a jury or electorate can and does agree. The question posed is absolutely essential.

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- The traditional conception of voting—the underlying theoretical foundation that goes back to 1299—is based on false premises : it is a faulty model of reality.

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- The model leads to an inconsistent theory : *real, unacceptable* paradoxes are unavoidable (e.g., Arrow's impossibility theorem, Condorcet's paradox).

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• So, why on earth continue to use it?

More generally, we claim that :

- The traditional methods of voting do not work : they often fail to elect the candidate sought by the voters.
- The traditional conception of voting—the underlying theoretical foundation that goes back to 1299—is based on false premises : it is a faulty model of reality.
- The model leads to an inconsistent theory : *real, unacceptable* paradoxes are unavoidable (e.g., Arrow's impossibility theorem, Condorcet's paradox).
- So, why on earth continue to use it?
- A new conception, a new model, a new theory with new methods are essential.

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• A more realistic conception of how voters imagine their "preferences"—their likes and dislikes—leads to a new model, a new theory and new method of voting.

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- A more realistic conception of how voters imagine their "preferences"—their likes and dislikes—leads to a new model, a new theory and new method of voting.
- It has been tested in voting, committee decisions and wine competitions.
- The majority judgement is natural, simple, robust, avoids unacceptable paradoxes, resists gaming, and—we claim—is the best of all known methods for choosing a winner and order of finish because it comes closest to meeting all the criteria of a good method of election.

The fundamental problem of electing and ranking : to find a *social decision function* (*SDF*) :

inputs \longrightarrow ouputs

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voters' "preferences" \longrightarrow society's "preferences"

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by which is meant

voters' rank-orderings \longrightarrow society's rank-ordering

First, this leads to serious inconsistencies.

Second, this is an <u>unrealistic vision</u> :

Voters <u>do not</u> have lists of candidates in their minds – the input messages are the wrong ones.

The fundamental problem of electing and ranking : to find a *social decision function* (*SDF*) :

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voters' "evaluations" \longrightarrow society's "evaluations"

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voters' grades $\longrightarrow \begin{cases} \text{society's grades} \\ \text{society's rank-ordering} \end{cases}$

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First, this leads to a consistent theory.

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$$\longrightarrow \begin{cases} \text{ society's grades} \\ \text{ society's rank-ordering} \end{cases}$$

First, this leads to a consistent theory.

Second, this is a much more $\underline{realistic\ vision}$ of what voters (and judges) have in mind.

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The great hope—since Ramun Llull in 1299—has been to choose a *Condorcet-winner* : a candidate who beats every possible opponent face-to-face.

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Of course, there may be no Condorcet-winner :

30%	32%	38%
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В	С	A
С	А	В

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30%	32%	38%		A	В	С
A	В	С	Α	-	68%	30%
В	С	A	В	32%	_	62%
С	A	В	С	70%	38%	_

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because

 $A(68\%) \succ B(62\%) \succ C(70\%) \succ A$

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The Condorcet Paradox (1786)

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The Condorcet paradox.

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In 1433, Nicolas Cusanus proposed what is known today as *Borda's method* (*1780*) :

Points	30%	32%	38%
2	A	В	С
1	В	С	Α
0	C	Α	В

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In 1433, Nicolas Cusanus proposed what is known today as *Borda's method* (*1780*) :

Points	30%	32%	38%	Borda score
2	A	В	С	A : 60+38=98
1	В	С	Α	B: 30+64=94
0	С	Α	В	C: 32+76=108

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Or,

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The *Borda-ranking* : $C \succ A \succ B$.

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First-past-the-post (UK, USA, ...) : A voter names one candidate (the input). The candidate most often named is elected (the output). The question asked to a voter is implicitly : which do you like best?

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First-past-the-post (UK, USA, ...) : A voter names one candidate (the input). The candidate most often named is elected (the output). The question asked to a voter is implicitly : which do you like best ?

Two-past-the-post (France, \ldots) : A voter names one candidate. If one candidate is named by more than 50% of the voters, he or she is elected. Otherwise, there is a run-off between the two candidates most often named.

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Approval voting permits a bit more information from voters. The voter may designate as many candidates as she wishes : which would you accept ?. The candidate most often designated wins.

AV was formally introduced by Robert Weber in 1977, though it seems to have been practiced in the Sparta of antique Greece...

Borda wrote :

I is generally accepted, and to my knowledge never challenged, that in an election the greatest number of votes always designates the will of the electorate... But I will show that this opinion, that is true when the election is between only two candidates , can mislead in all other cases... The thesis Traditional social choice Paradoxes in pactice Incon

Borda's example : the winner depends on the method

5%	33%	34%	28%
Α	Α	В	С
В	С	С	В
С	В	Α	Α

	A	В	С
A	-	38%	38%
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• (1) First-past-the-post : $A \succ B \succ C$

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Α	Α	В	С
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- (1) First-past-the-post : $A \succ B \succ C$
- (2)Two-past-the-post : $B \succ A \succ C$
- (3) Borda : $C \succ B \succ A$ (and Condorcet)

5%	33%	34%	28%
Α	Α	В	С
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Strategic manipulation pays :

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Strategic manipulation pays :

• If with (1), the 28% vote for B : B wins.

5%	33%	34%	28%
Α	Α	В	С
В	С	С	В
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	A	В	С
A	-	38%	38%
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- (3) Borda : $C \succ B \succ A$ (and Condorcet)

Strategic manipulation pays :

- If with (1), the 28% vote for B : B wins.
- If with (2), the 33% vote for C : C wins.

5%	33%	34%	28%
Α	Α	В	С
В	С	С	В
С	В	Α	Α

	A	В	С
A	-	38%	38%
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- (1) First-past-the-post : $A \succ B \succ C$
- (2)Two-past-the-post : $B \succ A \succ C$
- (3) Borda : $C \succ B \succ A$ (and Condorcet)

Strategic manipulation pays :

- If with (1), the 28% vote for B : B wins.
- If with (2), the 33% vote for C : C wins.
- If with (3), the 28% vote $B \succ C \succ A : B$ wins.

5%	33%	34%	28%
Α	Α	В	С
В	С	С	В
С	В	Α	Α

	A	В	С
A	-	38%	38%
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- (3) Borda : C wins.

Arrow's paradox :

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Α	Α	В	С
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A	-	38%	38%
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- (1) First-past-the-post : A wins
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- (3) Borda : C wins.

Arrow's paradox :

• If with (1), C (a loser) drops out, B wins; if B (a loser) drops out C wins.

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Α	Α	В	С
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Arrow's paradox :

- If with (1), C (a loser) drops out, B wins; if B (a loser) drops out C wins.
- If with (2), A (a loser) drops out, C wins.

33%	3%	16%	18%	30%
Α	В	Α	С	В
В	С	С	В	Α
С	Α	В	Α	С

	A	В	С
A	-	49%	79%
В	51%	_	66%
С	21%	34%	_

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33%	3%	16%	18%	30%
Α	В	Α	С	В
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The Borda-ranking is $A \succ B \succ C$.

33%	3%	16%	18%	30%		A	В	С
A	В	Α	С	В	Α	-	49%	79%
В	С	С	В	Α	В	51%	_	66%
С	Α	В	Α	С	С	21%	34%	_

The Borda-ranking is $A \succ B \succ C$.

Yet, B is the Condorcet-winner, $B \succ A \succ C$ the Condorcet-ranking.

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If C drops out :

B is the winner : Arrow's paradox !

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A reasonable method of voting should satisfy :

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- A reasonable method of voting should satisfy :
 - A1) Impartiality : Voters and candidates are treated equally.

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- A5) Strategy Proof : It is a dominant strategy to vote honestly.

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Unavoidable conundrum of the traditional model

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Unavoidable conundrum of the traditional model

When voters are asked to rank the candidates, then :

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Unavoidable conundrum of the traditional model

When voters are asked to rank the candidates, then :

Theorem (Arrow's impossibility)

No rule meets A1, A2, A3 and A4.
Unavoidable conundrum of the traditional model

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Theorem (Gibbard/Satterthwaite's impossibility)

No rule meets A1, A2, A3 and A5.

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- Paradoxes in pactice
- Incompatibility Between Electing and Ranking
- 5 Method of Majority Judgment
 - Wines
 - Elections
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- Theory of Majority Judgement
- Experimental evidences

8 Conclusions

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2000 Election	Votes	Electoral votes	Florida votes
George W. Bush	50,456,002	271	2,912,790
Albert Gore	50,999,897	266	2,912,253
Ralph Nader	2,882,955	0	97,488

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Arrow's paradox : a candidate's presence (having no chance of winning whatsoever) can change the outcome.

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First round results 2002 (16 candidates, 72% participation) :

Chir	ac	Le Pen	Jc	spin	Bayrou	Laguille	er <u>Chév</u>	ènement 🛛
19,88	%	16,86%	16	,18%	6,84%	5,72%	5	,33%
Mame	ère	Besanc	enot	Sain	t-Josse	Madelin	Hue	Mégret
5,25	%	4,25	%	4,	23%	3,91%	3,37%	2,34%
	(P	asqua)	Taub	oira	Lepage	Boutin	Gluckstei	in
		0%	2,32	%	1,88%	1,19%	0,47%	

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Le Per	<u>I</u> Jo	spin	Bayrou	Laguill	er <u>Chév</u>	ènement
16,86%	6 16	,18%	6,84%	5,72%	ó 5	,33%
e Besan	cenot	Sain	t-Josse	Madelin	Hue	Mégret
4,25	5%	4,	23%	3,91%	3,37%	2,34%
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0%	2,32	%	1,88%	1,19%	0,47%	
	E Le Per 16,86% e Besand 4,25 (Pasqua) 0%	$\frac{\text{Le Pen}}{16,86\%} \frac{\text{Jc}}{16}$ $e \text{Besancenot}}{4,25\%}$ $(\frac{\text{Pasqua}}{0\%}) \frac{\text{Taub}}{2,32}$	$\begin{array}{c} \underline{\text{Le Pen}} & \underline{\text{Jospin}} \\ 16,86\% & 16,18\% \end{array}$ $e \text{Besancenot} \text{Sain} \\ 4,25\% & 4, \end{array}$ $(\underline{\text{Pasqua}} & \underline{\text{Taubira}} \\ 0\% & 2,32\% \end{array}$	$\begin{array}{c c} \underline{\text{Le Pen}} & \underline{\text{Jospin}} & \text{Bayrou} \\ \hline 16,86\% & \overline{16,18\%} & 6,84\% \\ \hline \text{e} & \text{Besancenot} & \text{Saint-Josse} \\ 4,25\% & 4,23\% \\ \hline \hline (\underline{\text{Pasqua}}) & \underline{\text{Taubira}} & \text{Lepage} \\ \hline 0\% & 2,32\% & 1,88\% \\ \hline \end{array}$	Le PenJospinBayrouLaguill16,86%16,18%6,84%5,72%eBesancenotSaint-JosseMadelin4,25%4,23%3,91%(Pasqua)TaubiraLepageBoutin0%2,32%1,88%1,19%	Le PenJospinBayrouLaguillerChév16,86%16,18%6,84%5,72%5eBesancenotSaint-JosseMadelinHue4,25%4,23%3,91%3,37%(Pasqua)TaubiraLepageBoutinGluckstei0%2,32%1,88%1,19%0,47%

Second round results 2002 (80% participation) :

Chirac	Le Pen
82,21%	17,79%

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<u>c Le Pe</u>	<u>n</u> Jo	spin	Bayrou	Laguille	er <u>Chév</u>	ènement
6 16,86%	γ 1 6 ,	18%	6,84%	5,72%	, 5, 5,	33%
re Besan	cenot	Sain	t-Josse	Madelin	Hue	Mégret
4,2	5%	4,	23%	3,91%	3,37%	2,34%
(Pasqua)	Taub	ira	Lepage	Boutin	Gluckstei	n
0%	2,32	%	1,88%	1,19%	0,47%	
	<u>c Le Per</u> 6 16,869 re Besan 6 4,29 (<u>Pasqua</u>) 0%	$\begin{array}{c c} \underline{\text{Le Pen}} & \underline{\text{Jo}} \\ \hline & 16,86\% & 16, \\ \hline & & \\ \hline \hline & & \\ \hline \hline & & \\ \hline \hline \\ \hline & & \\ \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \\ \hline \hline$	$\begin{array}{c} \underline{\text{Le Pen}} & \underline{\text{Jospin}} \\ 6 & 16,86\% & 16,18\% \end{array}$ $\begin{array}{c} \overline{\text{re Besancenot Saint}} \\ 6 & 4,25\% & 4,1 \end{array}$ $(\underline{\text{Pasqua}} & \underline{\text{Taubira}} \\ 0\% & 2,32\% \end{array}$	$\begin{array}{c} \underline{\text{Le Pen}} \\ 6 \\ 16,86\% \\ \hline 16,18\% \\ \hline 16,18\% \\ \hline 6,84\% \\ \hline \\ 7e \\ 6 \\ 4,25\% \\ \hline 4,23\% \\ \hline \\ \hline \\ (\underline{\text{Pasqua}}) \\ \hline \\ 0\% \\ \hline \\ 2,32\% \\ \hline \\ 1,88\% \\ \hline \end{array}$		cLe PenJospinBayrouLaguillerChéve%16,86%16,18%6,84%5,72%5,reBesancenotSaint-JosseMadelinHue%4,25%4,23%3,91%3,37%(Pasqua)TaubiraLepageBoutinGlucksteir0%2,32%1,88%1,19%0,47%

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82,21%	17,79%	< 50% ?	> 50% ?

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82,21%	17,79%	< 50% ?	> 50% ?	> 75%	< 25%

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Arrow paradoxes and strategic manipulation galore !

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2007 presidential election.

In response to 2002, 30% of French voters did not vote for their first choices in 2007 : they voted strategically ("le vote utile").

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In response to 2002, 30% of French voters did not vote for their first choices in 2007 : they voted strategically ("le vote utile").

Minor candidates of the left obtained 27% in 2002, only 11% in 2007.

Minor candidates of the right obtained 16% in 2002, only 3% in 2007.

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2007 election, first round, 12 candidates

First round results :

Sarko	ozy	Royal	Bayrou	Le Pen	Besancenot	t de Villie	ers
31,18	3%	25,87%	18,57%	10,44%	4,08%	2,23%)
B	uffet	Voynet	Laguiller	· Bové	Nihous	Schivardi	
1	,93%	1,57%	1,33%	1,32%	1,15%	0,34%	

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2007 election, first round, 12 candidates

First round results :

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[Buffet	Voynet	Laguiller	Bové	Nihous	Schivardi		
	1,93%	1,57%	1,33%	1,32%	1,15%	0,34%		

Sarkozy was the winner with two-past-the-post system :

Nicolas Sarkozy 53% Ségoléne Royal 47%

Second round	poll of	March 20	(TNS-SOFRES) :
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	Bayrou	Le Pen	Royal	Sarkozy
Bayrou	—	84%	57%	54%
Le Pen	16%	—	25%	16%
Royal	46%	75%	—	46%
Sarkozy	46%	84%	54%	

These estimates confirmed by many polls and experiments.

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	Bayrou	Le Pen	Royal	Sarkozy
Bayrou	—	84%	57%	54%
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Second round poll of March 20 (TNS-SOFRES) :

These estimates confirmed by many polls and experiments. The same poll stated that

- among Royal voters, 72% for Bayrou against Sarkozy in the second round, and
- among Sarkozy voters, 75% for Bayrou against Royal in the second round.

	Bayrou	Le Pen	Royal	Sarkozy
Bayrou	—	84%	57%	54%
Le Pen	16%	—	25%	16%
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- among Royal voters, 72% for Bayrou against Sarkozy in the second round, and
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Bayrou the Condocet-winner and Borda-winner, Sarkozy the first-past-the-post-winner and two-past-the-post winner!!!

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Arrow's paradox : 1997 European Championships, men's free skating

	J_1	J_2	J ₃	J_4	J_5	J_6	J ₇	J ₈	J9	Mark	Place
Candeloro	3	2	5	2	3	3	5	6	6	3/5	2 nd
Kulik	2	4	2	3	6	5	3	4	5	4/6	5 th
Urmanov	1	1	1	1	1	2	1	1	1	1/8	1^{st}
Yagudin	4	3	3	6	4	6	4	3	2	4/7	4 th
Zagorodniuk	5	5	4	4	2	4	2	2	3	4/7	3 rd
Vlascenko	6	6	6	5	5	1	6	5	4	5/5	6 th

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Yagudin	4	3	3	6	4	6	4	3	2	4/7	4 th
Zagorodniuk	5	5	4	4	2	4	2	2	3	4/7	3 rd
Vlascenko	6	6	6	5	5	1	6	5	4	5/5	6 th

Before the performance of Vlascenko, the order was : 1st Urmanov, 2nd Zagorodniuk, 3rd Candeloro...!!!

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- The outcry over this flip-flop was so strident that the rules used for many years were changed. The ISU adopted the OBO rule ("one-by-one") in 1998 :
 - Rank the competitors by their number of wins (Condorcet's);
 - break any ties by using Borda's rule.

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- Namely, if voters are naturally restricted in their domain (because of some ideological opinions, such as a left right spectra), then it best avoids Arrow's paradox and combats strategic manipulations, among all raking-based methods.
- We prove it to be subject to Arrow's paradox, in a real skating competition (see our forthcoming OR paper).

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Accumulated evidence shows that judges in skating had strong national biases :

"The data suggests that countries are divided into two blocs, with the United States, Canada, Germany and Italy on one side and Russia, the Ukraine, France and Poland on the other" (Zitzewitz 2006).

The big "scandal" of the 2002 Olympic games in Salt Lake City : the first two finishers in the pairs figure skating competition.

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The big "scandal" of the 2002 Olympic games in Salt Lake City : the first two finishers in the pairs figure skating competition.

- The Russian pair first, the Canadian pair second ;
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- a French judge confessed having favored the Russians (under pressure) ... then denied it ;
- the decision amended : both pairs finished first ;
- deep divisions in the skating world leading to the formulation of a new system (replacing the newly adopted OBO rule).
- the new system is based only on evaluations and its supposed (without any theoretical support) to avoid Arrow's paradox and combat strategic manipulations. It does in some extend, but not optimally.

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Property 1 : the method is Condorcet consistent : the *Condorcet-winner* —(when he exists)—is always the first-ranked by the Condorcet-ranking.

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The *Condorcet-ranking* is the ranking that maximizes the Condorcet-score.

Property 1 : the method is Condorcet consistent : the *Condorcet-winner* —(when he exists)—is always the first-ranked by the Condorcet-ranking. Property 2 : computing a Condorcet-Kemeny ranking is NP-hard (Bartholdi, Tovey, and Trick, 1989).

Is Borda's method good for designating a winner, or a ranking, or both?

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Is Borda's method good for designating a winner, or a ranking, or both? Is Condorcet's method good for designating a winner, or a ranking, or both?

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- Given a method of ranking, the first-placed candidate is the winner.
- Given a method of designating a winner (or loser), he is the first-ranked (or last-ranked); the second-ranked is the winner among the remaining candidates; ...

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Given a profile of preferences, a *candidate-scoring method* assigns a nonnegative score to every candidate.

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• (1) assign a 0 to the worst possible candidate : give a 0 to a candidate last on every voter's list;

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Theorem

Borda-score characterization. The Borda-score is the unique candidate-scoring method that assigns a 0 to the worst possible candidate and correctly rewards minimal improvements.

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Moral : The Borda-score concerns winners.

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Given a profile of preferences, a *rank-scoring method* assigns a nonnegative score to every ranking.

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Given a profile of preferences, a *rank-scoring method* assigns a nonnegative score to every ranking.

The *opposite* of a ranking $A \succ B \succ C \succ D \succ \dots$ is $A \prec B \prec C \prec D \prec \dots$

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A rank-scoring method should :

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A rank-scoring method should :

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Given a profile of preferences, a *rank-scoring method* assigns a nonnegative score to every ranking.

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Theorem

Condorcet-score characterization. The Condorcet-score is the unique rank-scoring method that assigns a 0 to the worst possible order and correctly rewards minimal improvements.

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- (1) assign a 0 to the worst possible ranking : give a 0 to a ranking if every voter's preference is the opposite ranking;
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Theorem

Condorcet-score characterization. The Condorcet-score is the unique rank-scoring method that assigns a 0 to the worst possible order and correctly rewards minimal improvements.

Moral : The Condorcet-score concerns rankings.

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Are ranking and designating winners two sides of one coin?

∃ 990

Are ranking and designating winners two sides of one coin?

 $333: A \succ B \succ C \qquad 333: B \succ C \succ A \qquad 333: C \succ A \succ B$

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- By Borda :
 - A is the winner, B the loser : reasonable

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- By Borda :
 - A is the winner, B the loser : reasonable
 - Thus, society's order is $A \succ_S C \succ_S B$: unreasonable

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 - Thus, society's order is $A \succ_S C \succ_S B$: unreasonable
- By Condorcet :
 - $A \succ_S B \succ_S C$ and $C \succ_S A \succ_S B$ tied for first : reasonable

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No reasonable ranking function must choose $A \succ C \succ B$.

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No reasonable ranking function must choose $A \succ C \succ B$. Any reasonable choice function must choose $A \succ C \succ B$.

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No reasonable ranking function must choose $A \succ C \succ B$. Any reasonable choice function must choose $A \succ C \succ B$. A fundamental incompatibility between electing and ranking.

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- Paradoxes in pactice

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Method of Majority Judgment

- Wines
- Elections
- Strategy proof in raking
- Theory of Majority Judgement
- Experimental evidences

8 Conclusions

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Anjou	Bourgogne	Chablis	
Very good	Excellent	Excellent	
Very good	Very good	Excellent	
<u>Good</u>	<u>Good</u>	<u>Good</u>	
Good	Good	Passable	
Passable	Mediocre	Mediocre	

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 $\mathsf{Therefore}:\mathsf{Anjou}\succ\mathsf{Bourgogne}\succ\mathsf{Chablis}$

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The majority judgement : voters judge each candidate

Ballot : Election of the President of France 2007

To be president of France,

having taken into account all considerations, I judge, in conscience, that this candidate would be :

	Excellent	Very Good	Good	Acceptable	Poor	to Reject
Olivier Besancenot						
Marie-George Buffet						
Gérard Schivardi						
François Bayrou						
José Bové						
Dominique Voynet						
Philippe de Villiers						
Ségolène Royal						
Frédéric Nihous						
Jean-Marie Le Pen						
Arlette Laguiller						
Nicolas Sarkozy						

Check one single grade in the line of each candidate. No grade checked in the line of a candidate means to Reject the candidate.

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• Voters do not vote : they judge candidates in a *common language* of grades.

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- With 12 candidates, first- or two-past-the-vote allows a voter 13 or 39 (= 13 × 3) possible messages; the majority judgement with a common language of 6 grades allows a voter 6¹², *i.e.*, over 2 billion possible messages : voters can really express their opinions.

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- A candidate's set of grades determines his/her *majority-grade* : it is the "final-grade" conferred upon the candidate by the electorate.

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- A candidate's set of grades determines his/her *majority-grade* : it is the "final-grade" conferred upon the candidate by the electorate.
- The candidates are ranked according to their majority-grades : the first among them is the winner.

3

It is important to pose a clear and *solemn* question.

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By and large people—in particular, voters and judges—try to answer the question posed.

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In our new theory Arrow's impossibility theorem says : without a common language there can be no consistent collective decision.

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Common languages definitely do exist in practice (e.g., diving, gymnastic and figure skating competitions, wine competitions, students' grades, \dots).

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The words used in the French experiment constitute—for France—a common language. This is proven by extensive statistical analyses of the majority judgement ballots cast in the "2007 Orsay experiment."

3

The majority-grade

A candidate's majority-grade is the middlemost (or median) of his or her grades :

Excellent	Very Good	Good	Acceptable	Poor	To Reject
8%	23%	27%	12%	19%	11%

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8% + 23% + 27% = 58% of voters assign at least "Good"

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A majority of voters have assigned the candidate at least this grade,

8% + 23% + 27% = 58% of voters assign at least "Good"

A majority of voters have assigned the candidate at most this grade,

27% + 12% + 19% + 11% = 69% of voters assign at most "Good"

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Excellent	Very Good	Good	Acceptable	Poor	To Reject
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Excellent	Very Good	Good	Acceptable	Poor	To Reject
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Suppose a voter or bloc of voters judged this candidate "Very Good" : he/she or they have no reason to exaggerate the grade by giving an "Excellent."

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8%	23%	27%	12%	19%	11%

Suppose a voter or bloc of voters judged this candidate "Very Good" : he/she or they have no reason to exaggerate the grade by giving an "Excellent."

Suppose a voter or bloc of voters judged this candidate "Acceptable" : he/she or they have no reason to exaggerate the grade by giving a "Poor" or "to Reject."

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Theorem

The majority-grade is strategy-proof-in-grading

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The majority-grade respects a majority opinion : If a majority gives a grade of (say) "Acceptable" then that is the majority-grade.

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The majority-grade uniquely satisfies these and other properties.

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• Experiment conducted in 3 of Orsay's 12 voting bureaux.

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- These three are **not** representative of France.

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- Potential participants informed by mailings, local publications and posters with active participation of the Mayor's office.

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- After casting their official ballots, voters invited to cast majority judgment ballots (at adjacent tables and booths).

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- 2,360 voted officially, 1,752 (74%) participated in experiment, 1,733 ballots valid. 1,705 were different.

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- Experiment conducted in 3 of Orsay's 12 voting bureaux.
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- Carried out under identical conditions : ballots filled out in booths, inserted in envelopes and deposited in transparent urns.
- 2,360 voted officially, 1,752 (74%) participated in experiment, 1,733 ballots valid. 1,705 were different.
- Television interviews prove the satisfaction of voters.

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Results French Presidential elections, Orsay 3 Bureaux

A politically aware observer of France is able to fill in the blanks.

	Excel.	V.Good	Good	Accpt.	Poor	Rej.	Blank
	13.6%	30.7%	25.1%	14.8	8.4%	4.5%	2.9%
	16.7%	22.7%	19.1%	16.8%	12.2%	10.8%	1.8%
	19.1%	19.8%	14.3%	11.5%	7.1%	26.5%	1.7%
Voynet	2.9%	9.3%	17.5%	23.7%	26.1%	16.2%	4.3%
Besancenot	4.1%	9.9%	16.3%	16.0%	22.6%	27.9%	3.2%
Buffet	2.5%	7.6%	12.5%	20.6%	26.4%	26.1%	4.3%
Bové	1.5%	6.0%	11.4%	16.0%	25.7%	35.3%	4.2%
Laguiller	2.1%	5.3%	10.2%	16.6%	25.9%	34.8%	5.3%
Nihous	0.3%	1.8%	5.3%	11.0%	26.7%	47.8%	7.2%
Villiers	2.4%	6.4%	8.7%	11.3%	15.8%	51.2%	4.3%
Schivardi	0.5%	1.0%	3.9%	9.5%	24.9%	54.6%	5.8%
	3.0%	4.6%	6.2%	6.5%	5.4%	71.7%	2.7%

Red indicates the majority-grade.

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Results French Presidential elections, Orsay 3 Bureaux

A politically aware observer of France is able to fill in the blanks.

	Excel.	V.Good	Good	Accpt.	Poor	Rej.	Blank
Bayrou	13.6%	30.7%	25.1%	14.8	8.4%	4.5%	2.9%
Royal	16.7%	22.7%	19.1%	16.8%	12.2%	10.8%	1.8%
Sarkozy	19.1%	19.8%	14.3%	11.5%	7.1%	26.5%	1.7%
Voynet	2.9%	9.3%	17.5%	23.7%	26.1%	16.2%	4.3%
Besancenot	4.1%	9.9%	16.3%	16.0%	22.6%	27.9%	3.2%
Buffet	2.5%	7.6%	12.5%	20.6%	26.4%	26.1%	4.3%
Bové	1.5%	6.0%	11.4%	16.0%	25.7%	35.3%	4.2%
Laguiller	2.1%	5.3%	10.2%	16.6%	25.9%	34.8%	5.3%
Nihous	0.3%	1.8%	5.3%	11.0%	26.7%	47.8%	7.2%
Villiers	2.4%	6.4%	8.7%	11.3%	15.8%	51.2%	4.3%
Schivardi	0.5%	1.0%	3.9%	9.5%	24.9%	54.6%	5.8%
Le Pen	3.0%	4.6%	6.2%	6.5%	5.4%	71.7%	2.7%

Red indicates the majority-grade.

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Majority-grades, majority-gauges majority-ranking : French Presidential Elections, Orsay : Many voters

		Higher	The	Lower	Official	Ntnl
		M-G	M-G	M-G	vote	vote
3	Bayrou	44.3%	Good+	30.6%	25.5%	18.6%
2	Royal	39.4%	Good-	41.5%	29.9%	25.9%
1	Sarkozy	38.9%	Good-	46.9%	29.0%	31.2%
8	Voynet	29.8%	Acceptable-	46.6%	1.7%	1.6%
5	Besancenot	46.3%	Poor+	31.2%	2.5%	4.1%
7	Buffet	43.2%	Poor+	30.5%	1.4%	1.9%
10	Bové	34.9%	Poor-	39.4%	0.9%	1.3%
9	Laguiller	34.2%	Poor-	40.0%	0.8%	1.3%
11	Nihous	45.0%	To reject	-	0.3%	1.2%
6	Villiers	44.5%	To reject	-	1.9%	2.2%
12	Schivardi	39.7%	To reject	-	0.2%	0.3%
4	Le Pen	25.7%	To reject	-	5.9%	10.4%

Majority-gauge (p, α^{\pm}, q)

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Bayrou : (44.3%, Good+, 30.6%) > Royal : (39.4%, Good-, 41.5)

How could a voter who graded Royal above Bayrou manipulate? By lowering Bayrou's majority-gauge and raising Royal's.

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Bayrou : $(44.3\%, Good+, 30.6\%) \succ Royal : (39.4\%, Good-, 41.5)$

How could a voter who graded Royal above Bayrou manipulate? By lowering Bayrou's majority-gauge and raising Royal's.

Among voters who rated Royal above Bayrou : one who can lower Bayrou's majority-gauge cannot raise Royal's; one who can raise Royal's cannot lower Bayrou's.

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Bayrou : $(44.3\%, Good+, 30.6\%) \succ Royal : (39.4\%, Good-, 41.5)$

How could a voter who graded Royal above Bayrou manipulate? By lowering Bayrou's majority-gauge and raising Royal's.

Among voters who rated Royal above Bayrou : one who can lower Bayrou's majority-gauge cannot raise Royal's ; one who can raise Royal's cannot lower Bayrou's.

Theorem

No method is strategy-proof-in-ranking. The majority judgement is partially strategy-proof-in-ranking.

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Can exaggeration change the outcome? Yes, if *many* voters manipulate. But if only some 30% of those who can do so—and polls estimated that 30% of French voters cast votes not in accord with their convictions—they would have failed.

Contents

- The thesis
- 2 Traditional social choice
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- Incompatibility Between Electing and Ranking
- 5 Method of Majority Judgment
 - Wines
 - Elections
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- 6 Theory of Majority Judgement
- Experimental evidences

Conclusions

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Indeed, the decision mechanism used to operate markets itself uses a measure : money.

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Money plays the role of Language in the economy.

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Language plays the role of Money in the new theory of social choice.

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A problem is specified by a *profile* $\Phi = \Phi(C, \mathcal{J})$: an *m* by *n* matrix of grades assigned to the competitors (rows) by the judges (columns).

A *method of ranking* is a complete binary relation \succeq_S that, for a given profile Φ , compares any two competitors. It should possess certain minimal properties.

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- Axiom III transitive : $A \succeq_S B$ and $B \succeq_S C$ implies $A \succeq_S C$.
- Axiom IV independent of irrelevant alternatives : if $A \succeq_S B$ for the profile Φ then $A \succeq_S B$ for any profile Φ' obtained from Φ by eliminating or adjoining some other competitor (or row).

Social Ranking Functions

A method of ranking *respects grades* if the rank-order between two candidates A and B depends only on their sets of grades (i.e. the distribution of grades). Thus, It matters not which judge gave which grade.

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A method of ranking is neutral, anonymous, transitive and independent of irrelevant alternatives if and only if it is transitive, and respects grades.

A social ranking function (SRF) is a method of ranking that satisfies the four axioms.

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Suppose an electorate evaluated two candidates as follows :

	Good	Pass	Bad
X :	40%	35%	25%
Y :	35%	30%	35%

There is no doubt that X leads Y. What does a majority vote say?

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X wins with 65% That's correct.

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Y wins with 60%.

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Majority voting may fail even with 2 candidates!

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Axioms in Grading

An aggregation function is a function

 $f:\Lambda^n\to\Lambda$

judges' grades of one competitor \longrightarrow final grade of competitor f(exc., good, good, poor, v. good) = v.goodsatisfying :

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- anonymity : $f(..., \alpha, ..., \beta, ...) = f(..., \beta, ..., \alpha, ...);$
- *unanimity* : $f(\alpha, \alpha, \dots, \alpha) = \alpha$; and
- monotonicity :

$$\alpha_j \leq \beta_j \Rightarrow f(\alpha_1, \ldots, \alpha_j, \ldots, \alpha_n) \leq f(\alpha_1, \ldots, \beta_j, \ldots, \alpha_n)$$

and

$$(\alpha_1,\ldots,\alpha_n) \prec (\beta_1,\ldots,\beta_n) \Rightarrow f(\alpha_1,\ldots,\alpha_n) \prec f(\beta_1,\ldots,\beta_n).$$

Social Grading Functions

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A social grading function (SGF) f is a continuous method of grading that satisfies the 3 axioms.

The Game of Voting

The utility of a voter is some function $u_j(\mathbf{r}^*, \mathbf{r}, f, C, \Lambda)$ that may depend on many factors (the decision rule, the set of candidates, honesty, the set of messages, other's types and votes, etc).

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Given the mechanism and some private information, a voter chooses the message that maximizes his (unknown to us) utility function.

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We are going to prove that majority judgement is strategy-proof for a large class of utility functions. When it is not, it is shown that it combats manipulations in many well defined senses.

An aggregation function is *strategy-proof-in-grading* if

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- and if when a judge's honest input grade is some grade $r^- < r$, he cannot decrease the final grade.

Strategy-proof-in-grading implies it is a *dominant strategy* for a judge to honestly assign grades when his utility is single-peaked :

$$u_j = -|r_j^* - f(r_1, \ldots, r_n)|$$

The function that associates to a set of grades the *k*th highest grade is called the *k*th-order function f^k .

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If the mechanism is a point-summing method (the mean with respect to some parametrization), for almost all profiles, all voters can manipulate.

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A SGF is *strategy-proof-in-ranking* if for any two candidates A and B, if the final grade of A is below the final grade of $B : r^A < r^B$ and if some judge j has the opposed ranking $: r_j^A > r_j^B$,

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Strategy-proof-in-ranking implies it is a *dominant strategy* for a judge to honestly assign the grades whenever his utility function depends solely on the final ranking (or only on who is the winner).

Theorem (Extending Gibbard-Satterthwaite)

There exists no SGF that is strategy-proof-in-ranking.

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A SGF is *partially strategy-proof-in-ranking* if : for any two candidates A and B,

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- if j can increase A's final grade, he cannot decrease B's final grade.

Theorem

The unique SGFs that are partially strategy-proof-in-ranking are the order functions.

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Middlemost Aggregation Functions

The *middlemost* aggregation functions are (for $r_1 \ge \ldots \ge r_n$),

 $f(r_1,\ldots,r_n) = r_{(n+1)/2}$ when *n* is odd, and

 $r_{n/2} \ge f(r_1, \ldots, r_n) \ge r_{(n+2)/2}$ when *n* is even.

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Theorem

The unique aggregation functions that assign a final grade of r when a majority of judges assign r are the middlemost.

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Let $\lambda =$ probability a judge wishes to increase the final grade. The *probability* of effective-manipulability of f is

$$EM(f) = \max_{\mathbf{r}=(r_1,\ldots,r_n)} \max_{0 \le \lambda \le 1} \frac{\lambda \mu^+(f,\mathbf{r}) + (1-\lambda)\mu^-(f,\mathbf{r})}{n}.$$

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Theorem

The unique aggregation functions that minimize the probability of effective-manipulability are the middlemost. Point-summing-methods, f^1 and f^n maximize this probability.

More an order function is close to the middle, less it is manipulable.

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Minimizing Manipulability for SRF

A SRF is choice-monotone if $A \succeq_S B$ and one judge raises the grade he gives to A then $A \succ_S B$.

This is a natural idea that helps to resolve potential ties.

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A SRF is choice-monotone if $A \succeq_S B$ and one judge raises the grade he gives to A then $A \succ_S B$.

This is a natural idea that helps to resolve potential ties.

Theorem

The majority ranking is the unique choice-monotone, meaningful SRF that minimizes the probability of cheating and rewards consensus.

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BR-Majoritariane : for any candidate X and any strategy of a minority, the majority has a strategy that elects X.

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Examples : All reasonable methods : Borda, Condorcet, approval, 1- and 2-past-the-post, transferable-vote, majority-judgement.

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Theorem

Any candidate could be a Nash-equilibrium winner.

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 - Wines
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Conclusions

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- Words that carry meanings have a greater chance to be absolute than numbers
- With a numerical scale, voters assume the scores are summed, a clear invitation to manipulate—the greater the spread, the greater the opportunity.
- It must, in any case, be realized that adding numbers (or computing their averages) makes absolutely no sense unless the numbers belong to an *interval measure*.

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Percent of electors who used k (k = 1, ..., 6) grades :

No. grades :	1	2	3	4	5	6
percentages :	1%	2%	10%	31%	42%	14%

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Average numbers of each grade per ballot show the language was common :

	3	1 st	6 th	12 th	Samples of 100		Dsjt samples of 50	
	prcts.	prct.	prct.	prct.	Avg. (σ)	Rg	Avg. (σ)	Rg
Excll	0.7	0.7	0.7	0.7	0.7 (.07)	0.6/0.8	0.7 (.12)	0.5/0.9
V.Good	1.3	1.2	1.2	1.4	1.2 (.13)	1.1/1.5	1.3 (.16)	1.1/1.5
Good	1.5	1.5	1.4	1.6	1.5 (.13)	1.4/1.7	1.5 (.27)	0.9/1.8
Accp	1.7	1.7	1.7	1.8	1.8 (.15)	1.7/2.1	1.7 (.27)	2.1/2.6
Poor	2.3	2.3	2.3	2.2	2.3 (.19)	2.1/2.7	2.3 (.19)	2.1/2.6
Rjct	4.6	4.8	4.6	4.3	4.5 (.29)	4.1/4.8	4.5 (.41)	4.1/5.3

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Yet, the majority judgement winner not the same in all 3 precincts. Extensive statistical analyses of a large number of samples show the same stability.

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Grades :	Exclt	V Good	Good	Асср	Poor	Rejct
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Input messages that are voters' rank-orders are meaningless!

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From these 501 ballots random samples of 201 were drawn and the winners determined according to five different methods.

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10,000 random samples of 201 from 501 "representative" ballots, among only the \underline{three} principal candidates, number of wins :

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	$Left \leftarrow$		\rightarrow Right		
	Royal	Bayrou	Sarkozy	Tie	Cycle
First-past-the-post winner	656	0	9,261	83	-
Two-past-the-post winner	1,078	172	8,154	596	-
Approval <i>≿Very Good</i>	472	651	7,919	958	-
Majority judgement-winner	587	4,402	5,008	3	-
Condorcet-winner	138	8,390	954	389	129
Approval <i>≿Good</i>	36	9,436	30	498	-
Point-summing	132	9,444	260	164	-
Borda-winner	51	8,659	1,122	168	-

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	Royal	Bayrou	Sarkozy	Tie	Cycle
First-past-the-post winner	977	0	9,022	5	-
Two-past-the-post winner	1,146	98	8,197	559	-
Approval <i>≿Very Good</i>	467	658	7,947	928	-
Majority judgement-winner	606	4,326	5,065	3	-
Condorcet-winner	142	8,329	974	441	114
Approval <i>≿Good</i>	23	9,465	40	472	-
Point-summing	139	9,463	239	159	-
Borda-winner	12	9,976	0	12	-

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First- and two-past-the-post (unduly) penalize the centrist, point-summing and Borda (unduly) favor the centrist.

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Manipulability of methods : 10,000 random samples of 101 from 501 "representative" ballots, given that there is a same unique winner A and same unique runner-up B for every method.

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	Point-	Borda	First-	Approval	Approval	Cond-	Majority
	sum		p-p	\succeq Good	\succeq VGood	orcet	judge
Strat 1	9,965	9,313	8,699	8,569	8,407	7,042	6,142
Strat 2	9,769	7,864	4,411	8,849	8,557	4,641	5,313

Numbers of successful strategic manipulations :

Majority	Majority-	First	-past-the-		AV-
judgment	gauge	po	st score		score
1) Hollande	(45.1%, Good+, 43.3%)	1	28.6%	1	49.4%
2) Bayrou	(34.1%, Good-, 40.7%)	5	9.1%	3	39.20%
3) Sarkozy	(49.3%, Accept+, 39.6%)	2	27.3%	2	40.5%
4) Mélenchon	(42.5%, Accept+, 40.4%)	4	11.0%	4	39.1%
5) Dupont-Aignan	(40.6%, Poor+, 33.9%)	7	1.5%	8	10.7%
6) Joly	(36.8%, Poor-, 38.5%)	6	2.3%	6	26.7%
7) Poutou	(26.2%, Poor-, 45.7%)	8	1.2%	7	13.3%
8) Le Pen	(46.1%, Poor-, 47.6%)	3	17.9%	5	27.4%
9) Arthaud	(24.8%, Poor-, 49.9%)	9	0.7%	9	8.4%
10) Cheminade	(48.0%, to Reject, -)	10	0.4%	10	3.2%

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Only the five major candidates were tested.

Condorcet- ranking	Hollande	Bavrou	Sarkozv	Mélenchon	Le Pen	Borda- ranking
1) Hollande	_	51.6%	53.9%	68.5%	64.1%	1) 59.5%
2) Bayrou	48.4%	-	56.5%	59.4%	70.5%	2) 58.7%
3) Sarkozy	46.1%	43.5%	_	50.5%	65.7%	3) 51.4%
4) Mélenchon	31.5%	40.6%	49.5%	-	59.7%	4) 45.3%
5) Le Pen	35.9%	29.5%	34.3%	40.3%	-	5) 35.0%

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- The MJ-, Condorcet- and Borda-rankings are identical (put Bayrou comfortably ahead of Sarkozy) but differ from first-past-the-post and AV (place Bayrou behind).
- The margin of victory and ranking are much comfortable with MJ than all other methods. More information is better.

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4) Mélenchon	31.5%	40.6%	49.5%	-	59.7%	4) 45.3%
5) Le Pen	35.9%	29.5%	34.3%	40.3%	-	5) 35.0%

- The MJ-, Condorcet- and Borda-rankings are identical (put Bayrou comfortably ahead of Sarkozy) but differ from first-past-the-post and AV (place Bayrou behind).
- The margin of victory and ranking are much comfortable with MJ than all other methods. More information is better.

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Only the five major candidates were tested.

Condorcet- ranking	Hollande	Bayrou	Sarkozy	Mélenchon	Le Pen	Borda- ranking
1) Hollande	-	51.6%	53.9%	68.5%	64.1%	1) 59.5%
2) Bayrou	48.4%	-	56.5%	59.4%	70.5%	2) 58.7%
3) Sarkozy	46.1%	43.5%	-	50.5%	65.7%	3) 51.4%
4) Mélenchon	31.5%	40.6%	49.5%	-	59.7%	4) 45.3%
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- Other experiments confirm all results (forthcoming OR paper).

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Field experiment. French Socialist primaries, Fresnes, 2011

Majority judgment	Majority-gauge	First-past-the-post
1 Hollande	(18.2%, <i>Excellent</i> -, 49.7%)	35.7%
2 Aubry	(48.5%, Very Good+, 20.2%)	34.5%
3 Montebourg	(33.7%, Very Good –, 39.1%)	18.5%
4 Royal	(37.5%, Good-, 38.9%)	6.0%
5 Valls	(36.4%, Good-, 40.4%)	5.3%
6 Baylet	(27.2%, Acceptable-, 48.2%)	0.0%

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Condorcet-							Borda-
ranking	Aubry	Hollande	Montebourg	Royal	Valls	Baylet	ranking
1 Aubry	-	50.2%	68.5%	85.0%	85.9%	95.5%	77.0%
2 Hollande	49.8%	-	65.3%	85.4%	87.1%	94.8%	76.5%
3 Montebourg	31.5%	34.7%	-	68.3%	69.0%	91.8%	59.1%
4 Royal	15.0%	14.6%	31.7%	-	54.7%	78.2%	38.1%
5 Valls	14.1%	12.9%	31.0%	45.3%	_	78.9%	36.4%
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2 Hollande	49.8%	-	65.3%	85.4%	87.1%	94.8%	76.5%
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Practice shows what was illustrated in theory : majority voting can elect a candidate who is not judged to be the best according to the evaluations.

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- 3 Paradoxes in pactice
- Incompatibility Between Electing and Ranking
- 5 Method of Majority Judgment
 - Wines
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- Theory of Majority Judgement
- Experimental evidences

8 Conclusions

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- Close scores occur often to be important : the 2000 U.S. presidential race was decided by an official margin of 537 votes.

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- To prove it satisfies these requirements, instead of enlarging voters' possible expressions of opinion, it must restrain them (implicitly assuming that voters' rank-order inputs are naturally expressible along a clear-cut ideological ordering of the candidates).
- It cannot be the case in most competitions (wine, skating) and there is ample experimental evidence that shows voters do not behave in accord with this restriction.

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- On the other hand, in a non-polarized election such as a primary, often many candidates end with strong "majorities," so again none has a really clear and distinctive mandate.
- In both cases experience shows that scores may be close, so the results are all the more manipulable.

Field experiment. French Socialist primaries, Alfortville, 2011.

Majority	Majority-	Approval	Approval	Reported	Actual
judgment	gauge	voting	judgment	votes	votes
1 Hollande	(40.1%, Good+, 25.4%)	87.3%	87.0%	37.7%	39.7%
2 Aubry	(33.1%, Good+, 30.6%)	85.2%	82.0%	29.2%	28.9%
3 Montebourg	(39.8%, Accept.+, 36.3%)	64.1%	63.7%	12.5%	12.3%
4 Valls	(28.5%, Accept, 44.7%)	53.2%	55.3%	10.0%	8.6%
5 Royal	(27.1%, Accept, 47.2%)	53.5%	52.8%	10.3%	9.7%
6 Baylet	(41.7%, Poor+, 28.9%)	25.7%	20.4%	0.4%	0.7%

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Darwin would conclude : for a better democracy, natural selection will lead, in the long term, to majority judgment.

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