Dynamic Matching Markets with Limited Availability

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Availability at oDesk

oDesk
Love the way you work.

The Basics

When clients send out invitations and freelancers don't reply, it's a frustrating experience that makes those clients less likely to hire anyone. So, beginning in November, your profile will show a responsiveness indicator. This creates a system where freelancers who reply to invites - either accepting or declining - will be more likely to receive future invites.

Profiles will show one of these indicators:

- **Replies within a day** ● You accept or decline most invitations within the first day
- **Replies within a 3 days** ● You accept or decline most invitations within 72 hours
- **Replies Sometimes** ● You do respond to many invitations, but not as often or as quickly
- **Replies Rarely** ● You don't respond to most invitations

If you have received too few invitations recently to calculate a score, your profile will not include a responsiveness indicator.

Some Observations

1. Markets like oDesk are dynamic and asynchronous, with agents arriving and departing intermittently.
2. Agents on the other side may be unavailable to you.
3. You spend time and effort evaluating others before learning whether they are available.
4. Submitting a request or application is relatively easy.
5. There is a central operator who is able to observe and regulate the market.

Motivating Questions

1. How does the fact that availability is unobservable affect the market?
2. What can operator do to improve market outcomes?

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A *dynamic, two-sided, one-to-one* matching market with *homogeneous* agents.

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Agent Arrival

- Buyers arrive at rate n.
- They stay in the system for one time unit.
- Sellers arrive at rate rn.
- Upon arrival, sellers apply to each buyer in the market independently with probability m/n.
- Sending each application costs c_a.

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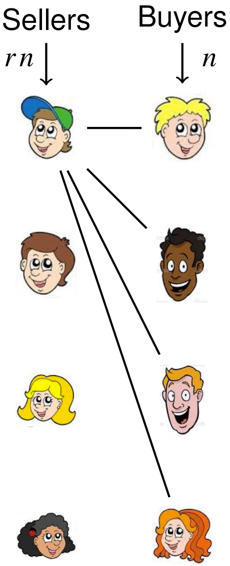

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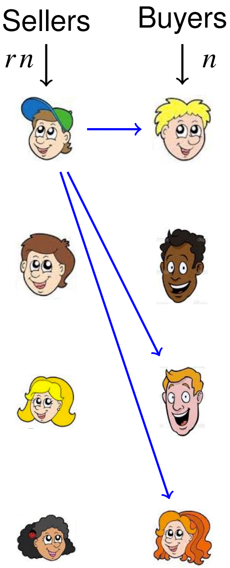

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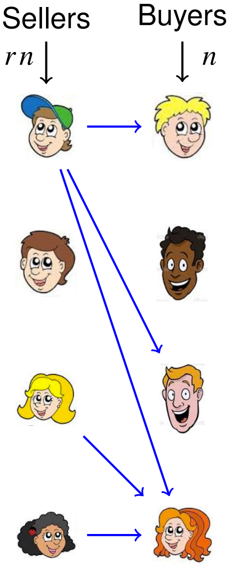
Agent Departure

- Upon exit, buyers may screen applicants.
- Screening an applicant costs c and reveals their fitness.
- Each seller is qualified with probability β.
- Buyers screen before making an offer.
- Sellers respond immediately to offers.



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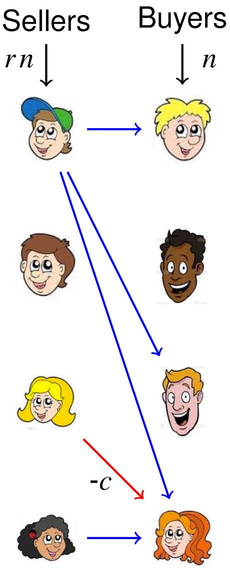

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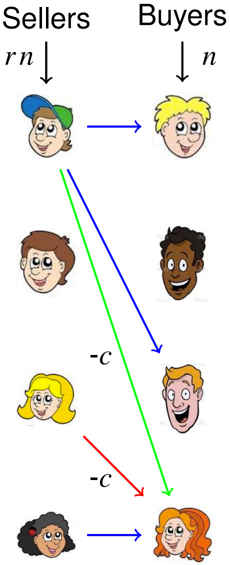


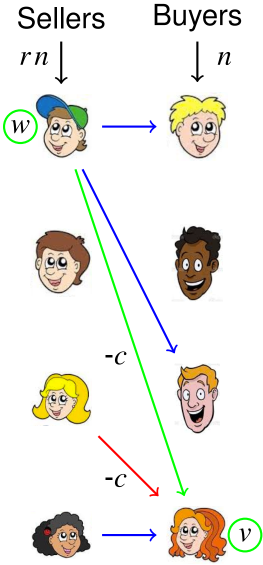
The Model

Match Surplus

A successful match generates a surplus of v for the buyer and w for the seller.

Thus buyer surplus is:
$v \cdot \mathbf{1}(\text{Hires Successfully}) - c \cdot (\text{\# Screened})$.

And seller surplus is:
$w \cdot \mathbf{1}(\text{Gets Hired}) - c_a \cdot (\text{\# Applications})$.



Additional Notes

- Seller always accept the first offer.
- Hired sellers are unavailable to other buyers (but the buyers don't know it).

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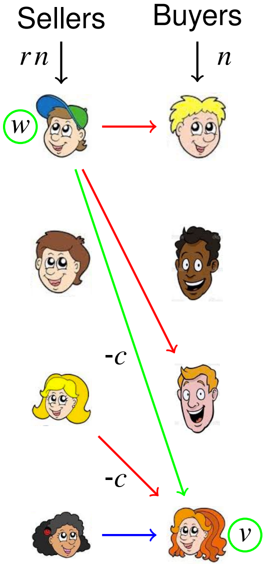

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A Mean Field Model

We consider a mean field model inspired by a regime where $n \to \infty$ – i.e., where buyer and seller arrival rates become large.

Mean field assumptions:

1. Each seller assumes that each application yields an offer with probability p (i.i.d.).
2. Each buyer assumes that each applicant is available with probability q (i.i.d.).

A Mean Field Model: Optimality

Optimal strategies:

1. Can show that a fixed p induces an optimal choice of m (application intensity).
2. For buyers, can show that a fixed q induces an optimal strategy that mixes between simple sequential screening (with prob. α) and exiting (prob. $1 - \alpha$).

[Simple sequential screening: buyer screens each applicant one at a time, and makes an offer to the first compatible applicant (if any).]

A Mean Field Model: Consistency

Given m and α, what p and q result in the market?

- Suppose a seller applies to k buyers. The probability that s is available when screened by a given buyer is:

$$\frac{1}{k}\sum_{j=0}^{k-1}(1-p)^j = \frac{1-(1-p)^k}{pk}.$$

- Each seller sends a Poisson(m) number of applications in the mean field limit.
- Averaging over # of applications sent yields:

$$q = \frac{1-e^{-mp}}{mp}. \tag{1}$$

A Mean Field Model: Consistency

Given m and α, what p and q result in the market?

- Suppose a given seller applied to a buyer with ℓ competing applicants; what is the probability this buyer screens the seller?
$$\frac{\alpha}{\ell+1}\sum_{j=0}^{\ell}(1-\beta)^j = \frac{\alpha(1-(1-\beta)^{\ell+1})}{\ell+1}.$$
- Number of competing available applicants is Poisson(rmq) in the mean field limit.
- Averaging over # of competitors yields:

$$p = \frac{\alpha(1-e^{-rm\beta q})}{rmq}. \qquad (2)$$

We show: *Given m and α, there exists a unique p and q solving* (1)-(2)*.*

Mean Field Equilibrium

1. **Optimality**: Given p and q, find optimal seller response m and buyer response α.
2. **Consistency**: Given m and α, find p and q that would result in a steady state of the resulting market ("mean field steady state").

A *mean field equilibrium* is a fixed point of the composed map.

Theorem

Mean field equilibrium exists and is essentially unique.

Nice Properties of Mean Field Equilibria

1. Mean field equilibria exist and are unique
2. Appealingly simple strategies
3. Justified as an ε-Bayes-Nash equilibrium in large markets
4. Tractable analysis

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- Sellers choose an expected number of applications.
- Buyers choose whether to bother screening.

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Basically means that the mean-field independence assumptions hold as the market grows large.

We prove this using a contraction argument on the process describing the sellers in the system.

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What happens without intervention?

Key statistic: "Normalized" screening cost $c' = \frac{c}{\beta v}$.

Theorem (Performance of Unregulated Market)

If $c' > \frac{1}{r \ln\left(\frac{r}{r-1}\right)}$*, then as* $c_a \to 0$*,*

- *Buyer surplus converges to zero.*
- *Seller surplus converges to* $w(1-e^{-\gamma})(1-e^{-\gamma}/c')$*, where* $(1-e^{-\gamma})/\gamma = c'$*.*

Otherwise,

- *Buyer surplus converges to* $v\left(1 - c' r \ln\left(\frac{r}{r-1}\right)\right)$*.*
- *Seller surplus converges to* $\frac{w}{r}\left(1-(r-1)\ln\left(\frac{r}{r-1}\right)\right)$*.*

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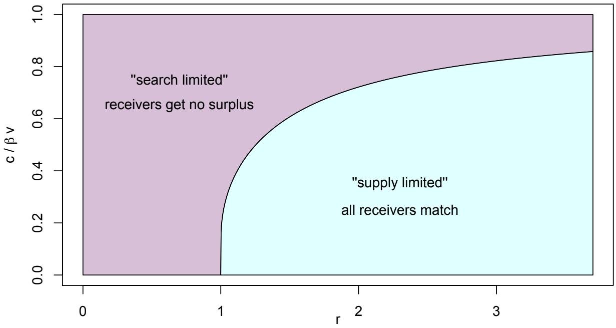

Two Regimes: “supply limited" and "search limited"

- In one regime, the number of matches is limited by the number of buyers in the marketplace.
- In the other regime, the number of matches is limited by the screening cost.

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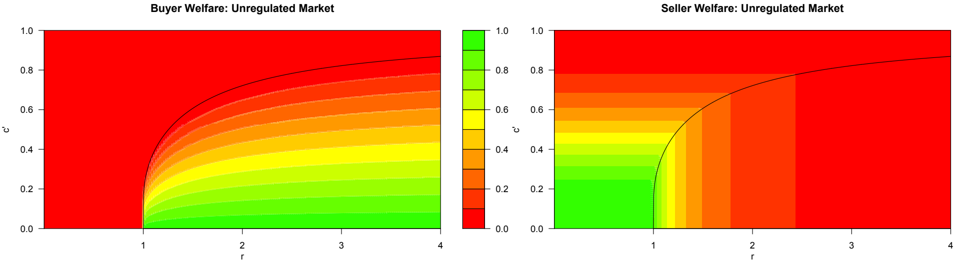

Some Problems

- In the “search-limited" regime, buyers get zero surplus. This holds whenever $r \leq 1$, even if c' is very small.
- In this regime, agents on both sides remain unmatched, and adding more buyers will not help sellers.
- In general, sellers lose much of their potential surplus to application costs, even though $c_a \to 0$ (for $r = 1.4$, sellers get less than half of w/r).

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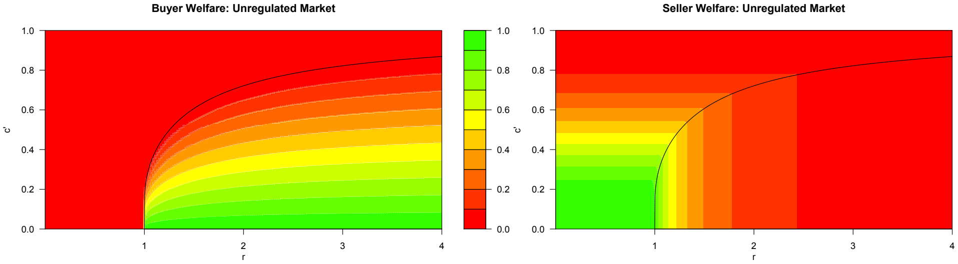

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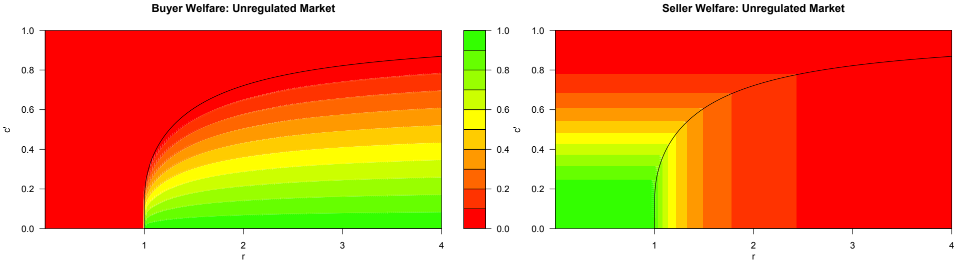

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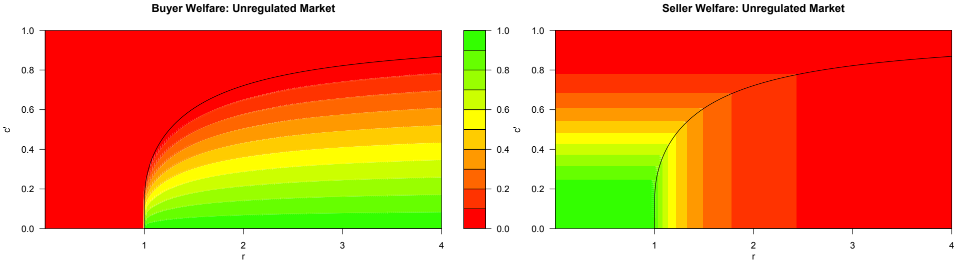

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What's going on, and can we help?

The Problem

Sellers are over-applying. When a seller sends an extra application, they generate externalities that harm

- Other sellers (who face more competition).
- Other buyers to whom the seller applies (who are now less likely to get them).

A Possible Solution

Buyer welfare is: $v \cdot \mathbf{1}(\text{Hires Successfully}) - c \cdot (\text{\# Screened})$.
Seller welfare is: $w \cdot \mathbf{1}(\text{Gets Hired}) - c_a \cdot (\text{\# Applications})$.

Restricting the sending of applications trades off screening and application costs against number of matches formed.

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Restricting the sending of applications trades off screening and application costs against number of matches formed.

How much benefit can we provide? (using math)

Theorem (Performance of the Regulated Market: Buyers)

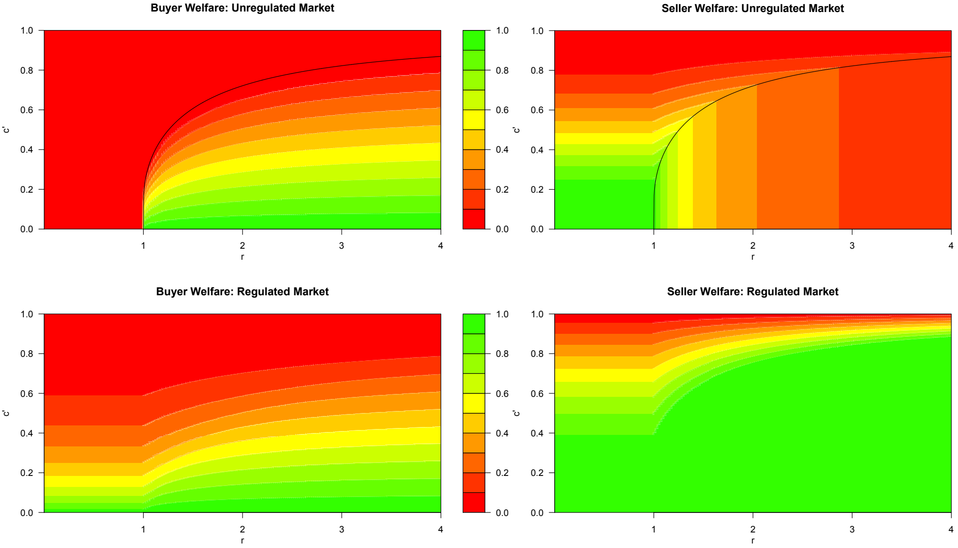
- *When* $c' < \frac{r-1}{r}$*, for any application limit* $\overline{m}$*, the unregulated market is superior for buyers for sufficiently small* c_a*.*
- *Otherwise, for an appropriate choice of* $\overline{m}$*, buyer welfare converges to* $vr(1-c'+c'\log c')$*, and seller welfare to* $w(1-c')$*.*

Theorem (Performance of the Regulated Market: Sellers)

Seller welfare is always improved by moderately restricting m*.*

- *When* $c' > 1/(r\ln\left(\frac{r}{r-1}\right))$*, if* $\overline{m}\to\infty$*,* $c_a\overline{m}\to 0$*, then seller welfare approaches* $w(1-e^{-\gamma})$
- *When* $c' \leq 1/(r\ln\left(\frac{r}{r-1}\right))$ *if* $\overline{m}\to\infty$*,* $c_a\overline{m}\to 0$*, then seller welfare approaches* w/r*.*

How much benefit can we provide? (using pictures)



Parting Thoughts

- Our work does not include two effects that may be pertinent in practice.
  - *Wages*: Recent work by Kircher suggests that with endogenous wages but without screening costs, a form of "constrained" efficiency can be achieved. What happens in a model with endogenous wages and screening?
  - *Asymmetry*: In our model sellers care about compatibility but buyers do not. What happens in a model where both care about compatibility?
- We do not model the fact that if people can only send a small number of applications, they/the system will contact agents with whom they are most likely to be compatible.
  - This suggests that the benefits of restriction may be even greater than estimated.
  - Of course, if marketplaces could just not show people that would reject you, life would be better. We show that even withholding random agents might be a good idea.

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