Mixed Integer Nonlinear Programming Applied To Dike Height Optimization

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Successful MINLP application

Finalist Edelman Award, April 2013

TILBURG UNIVERSITY
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TU Delft
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CPB Netherlands Bureau for Economic Policy Analysis

Rijkswaterstaat
Ministry of Infrastructure and the Environment

Deltares
Enabling Delta Life

HKV
LIJN IN WATER

Outline

1. Background
2. Model
3. Implementation
4. Results and conclusions

Dikes In The Netherlands

- Total length of dikes: 3500 kms
- Protection against flooding (sea, rivers, lakes)
- 55% of the Netherlands is below sea level
- Total expenses: 1 billion euros per year

1953: Flood in Zeeland

Soon after 1953

- Delta Committee installed

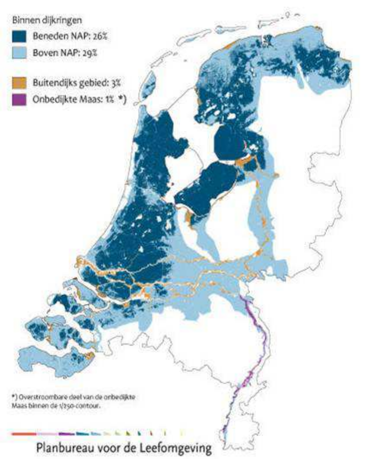
Cost-benefit-analysis

Van Dantzig determined optimal dike heights by looking at

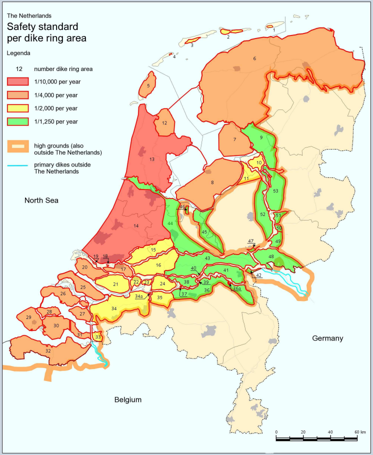
- **Costs:** Investments in heightening dikes (not just regular maintenance)
- **Benefits:** Reduced risk of damage as a result of flooding

See Econometrica (1956).

Flood prone area



Current safety standards



Project Background

Recent Developments

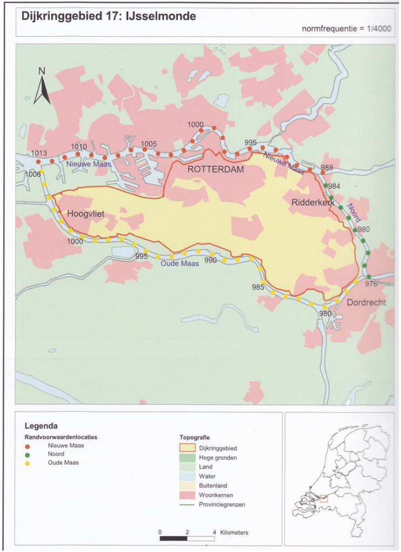
- 1993 & 1995: critical situation in many areas, 200,000 people were evacuated
- 2008: Second Delta Committee report
  - adviced to increase safety standards by a factor 10!
- Delta Programme initiated
  - foster the protection against high water now and in the future

Our project

- Project initiated by Deltares (research institute specialized in water issues)
- Extend Eijgenraam's improvement of Van Dantzig's cost-benefit analysis to non-homogeneous dike rings
- Goal: define new safety standards (to be incorporated in the law)

Outline

Dike Ring



- A dike ring protects a certain area of land against flooding
- Consist of several segments such as dikes, dunes, structures
- Each segment has different characteristics
  - flood probability
  - rise of water level
  - investment costs

Flood Probability (1)

Segment flood probability (per year) at time t

$$P_{\ell t} = P_{\ell 0} \exp\big(\alpha_\ell(\eta_\ell t - h_{\ell t})\big)$$

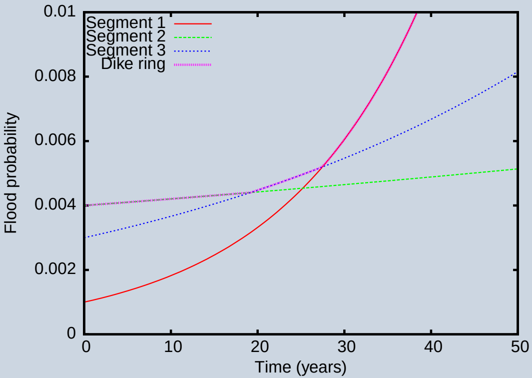
- $P_{\ell 0}$: initial flood probability segment ℓ
- α_ℓ: parameter exp. distr. for extreme water levels (1/cm)
- η_ℓ: structural increase of water level (cm/year)
- $h_{\ell t}$: height segment ℓ at time t

Flood Probability (2)

Dike Ring

Probability that there is a flood is the maximum of the segment probabilities:

$$P_t = \max_{\ell} P_{\ell t}$$



Damage Costs

Damage costs if a flood occurs at time t

$$V_t = V_0 \exp\left(\gamma t + \zeta \min_{\ell} h_{\ell t}\right)$$

- V_0: initial damage
- γ: dike ring wealth growth rate (1/year)
- ζ: loss increase due to absolute height of dike (cm/year)

Expected Damage at time t

$$S_t = P_t V_t = \max_{\ell} S_{\ell 0} \exp\left(\beta_\ell t - \alpha_\ell h_{\ell t} + \zeta \min_{\ell'} h_{\ell' t}\right)$$

with $S_{\ell 0} = P_{\ell 0} V_0$ and $\beta_\ell = \alpha_\ell \eta_\ell + \gamma$

Investment Costs

Investment costs determined by

- the current height $h^-_{\ell t}$ at time t (before heightening)
- the size of the heightening $u_{\ell t}$

Height at time t after heightening: $h_{\ell t} = h^-_{\ell t} + u_{\ell t}$

Exponential costs

$$I_\ell(h^-, u) = \begin{cases} (\phi_{\ell 0} + \phi_{\ell 1} u) \exp\big(\phi_{\ell 2}(h^- + u)\big) & \text{if } u > 0 \\ 0 & \text{if } u = 0 \end{cases}$$

Quadratic costs

$$I_\ell(h^-, u) = \begin{cases} \phi_{\ell 0} + \phi_{\ell 1} u + \phi_{\ell 2}(h^- + u)^2 & \text{if } u > 0 \\ 0 & \text{if } u = 0 \end{cases}$$

Decision Problem

- Choose timing and size of segment heightenings

$$0 = t_0 < t_1 < t_2 < \dots$$

$$h_{\ell t} = h_{\ell 0} + \sum_{i=0}^{k} u_{\ell k}, \qquad t_k \leq t < t_{k+1}$$

- … to minimize the sum of the discounted expected damage costs and discounted investment costs

$$\int_0^{\infty} S_t e^{-\delta t} dt + \sum_{\ell} \sum_{k=0}^{\infty} e^{-\delta t_k} I_\ell \Big(h_{\ell 0} + \textstyle\sum_{i=0}^{k-1} u_{\ell i}, u_{\ell k} \Big)$$

- Evaluating the integral (for optimization purposes) can only be done by an approximation

MINLP Model (1)

- Finite planning horizon: $[0, T]$.
- Discretization of planning horizon:

$$0 = t_0 < t_1 < \cdots < t_K < t_{K+1} = T$$

Interval sizes $t_{k+1} - t_k$ not necessarily equidistant.

- Binary decision variables

$$y_{\ell k} = \begin{cases} 1 & \text{if segment } \ell \text{ is heightened at time } t_k, \\ 0 & \text{otherwise.} \end{cases}$$

- Constraint: $u_{\ell k} \leq M y_{\ell k}$, where M is larger than the largest possible dike heightening.

MINLP Model (2)

After solving some technical issues we end up with the following MINLP

$$\min_{u_{\ell k}, h_{\ell k}, y_{\ell k}} \sum_{k=0}^{K} \left\{ \sum_{\ell=1}^{L} e^{-\delta t_k} \left(\phi_{\ell 0} y_{\ell k} + \phi_{\ell 1} u_{\ell k}\right) e^{-\phi_{\ell 2} \sum_{i=0}^{k} u_{\ell k}} \right.$$

$$\left. + \max_{\ell} \frac{S_{\ell 0}}{\beta_{1\ell}} \exp\left\{\zeta h_{\ell * k} - \alpha_\ell h_{\ell k}\right\} \left[e^{\beta_1 t_{k+1}} - e^{\beta_1 t_k}\right] \right\}$$

$$+ \max_{\ell} \frac{S_{\ell 0}}{\delta} \exp\left\{\zeta h_{\ell * T} + \beta_{1\ell} T - \alpha_\ell h_{\ell T}\right\}$$

subject to

$$0 \leq u_{\ell k} \leq M y_{\ell k} \quad \forall \ell, k$$

$$h_{\ell k} = \sum_{i=0}^{k} u_{\ell k} \quad \forall \ell, k$$

$$y_{\ell k} \in \{0, 1\} \quad \forall \ell, k$$

Problem characteristics

- Number of segments L: 2–10
- Number of intervals K: $\pm$30 (5 to 10 year intervals)
- $(K + 1)L$ binary decision variables (timing of update)
- $(K + 1)L$ continuous decision variables (size of update)
- Several auxiliary variables (from rewriting the objective)
- The problem is not convex, but has many nice convexity properties
  - for the quadratic investment cost function it can be written as a convex problem

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Main tools

Environment

Model implemented in AIMMS: integrated combination of a modeling language, a graphical user interface, and numerical solvers

Solvers

AOA: outer approximation method for MINLP

- Iteratively solve NLP and MIP models to approximate the original MINLP
- Method designed for convex optimization problems
- In our case: better and faster than a global method (e.g. BARON) even though our model is not completely convex
- CONOPT and CPLEX used for subproblems

Practical Difficulty

Problem size determined by

1. Number of segments L (fixed)
2. Discretization of planning horizon K: also important for
   - richness of possible solutions
   - approximation of expected damage

Curse of dimensionality

- Solution time for large instances is too high
- In practice: solving instances with $L > 6$ and a reasonable value for K becomes problematic
- We need a way to speed up the optimization process

Idea

- Instead of solving the original MINLP with the desired discretization we can do something different

Iterative method

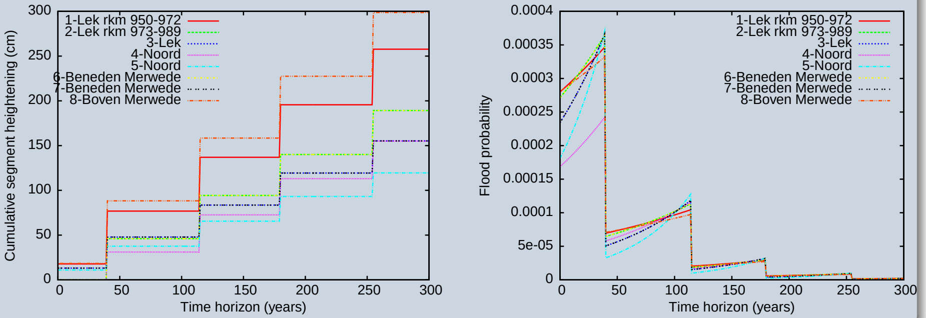
1. Quickly find a set of reasonable solutions
2. Choose the best solution and try to improve even further

How to find a reasonable solution quickly?

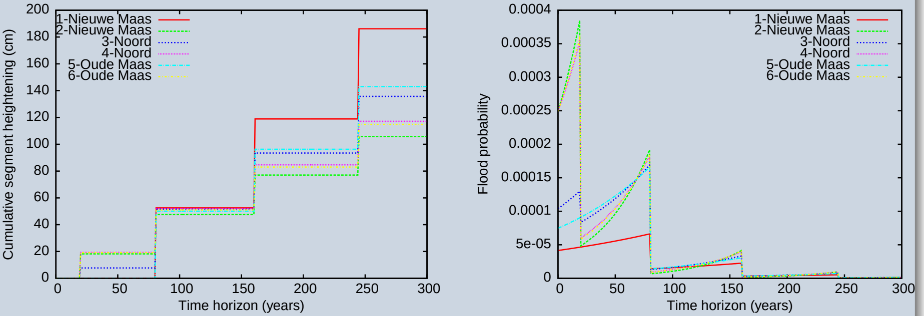
- Start with a rough discretization
- Use common solution structures

Common Solution Structures (1)

All segments heightened simultaneously (except at $t = 0$)



Many segments heightened simultaneously



- Two segments (1 and 5) are not updated together with the other segments at $t = 20$.

Enforce Solution Structure

- Extend the MINLP formulation with constraints that enforce a solution structure
- This reduces the feasible region and speeds up the solution process

Constraints

- **All segments heightened simultaneously**

$$y_{1k} = y_{\ell k}, \qquad \forall \ell,\ k \geq k_s$$

- **Subset of segments heightened simultaneously**

$$y_{\ell' k} = y_{\ell k}, \qquad \forall \ell \in G, k \geq k_s$$

for a cleverly chosen subset G of all segments (based on segment characteristics)

Iterative Method

Algorithm

1. Set rough discretization
2. Solve model with different solution structures
3. Choose the best solution (structure)
4. Refine discretization (in interesting neighborhoods)
5. Resolve best solution structure

Benefits

- Finer discretization than possible in original formulation
- Gives very good solutions at a fraction of the solution time of original MINLP!
- Solution times
  - Without iterative method: several hours or even "infinite"
  - With iterative method: 1–60 minutes

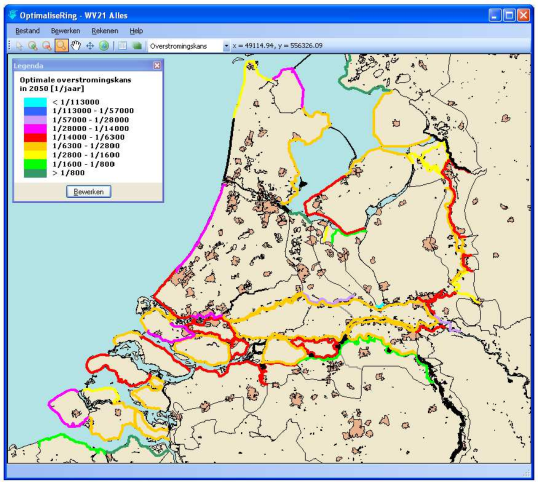
Outline

Scientific reports

- Ruud Brekelmans, Dick den Hertog, Kees Roos, and Carel Eijgenraam. Safe Dike Heights at Minimal Costs: The Nonhomogeneous Case. *Operations Research* November/December 2012 60:1342-1355
- Carel Eijgenraam, Ruud Brekelmans, Dick den Hertog and Kees Roos. Flood Prevention by Optimal Dike Heightening. *Working paper*. Under revision at Management Science

Implementation in OptimaliseRing

MINLP model and solver have been implemented by HKV

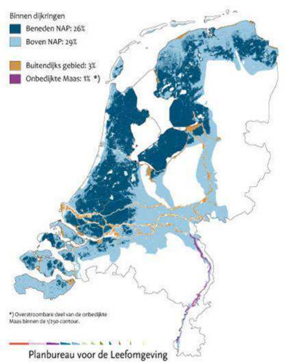


Analysis of all dike rings

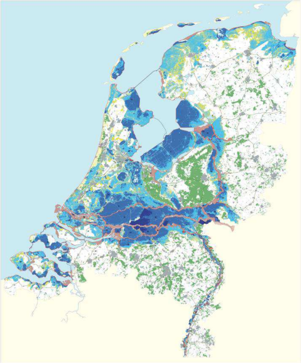
Current safety standards are OK, except for three areas:

- River area
- Southern Flevoland
- Parts of Rijnmond-Drechtsteden and Voorne Putten.

Flood prone area



Maximum water depths



Decision process

- Results summarized in report by Deltares (november 2011)
  - Factor 10 increase in safety standards not necessary: investment costs 11 billion euro
  - New recommendation: investment costs 3.5 billion euro
- Report has been discussed in House of Parliament and "Adviescommissie Water" (headed by His Royal Highness, Prince Willem van Oranje)
- Vice Minister decided according to recommendations in the report (letter dated May 7, 2012)
- Final safety standards will be stated in Dutch Water Act in 2017.

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- The continuous improvement in (MINLP) solvers creates new possibilities that wouldn't be possible several years ago

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  - . . . you need to help the solvers where you can

Conclusions

- The continuous improvement in (MINLP) solvers creates new possibilities that wouldn't be possible several years ago
- But, even today ...
  - ...it is necessary to come up with the right model formulation to solve your problem
  - ...you need to help the solvers where you can
- Don't be afraid of MINLP!

Dike height optimization in The Netherlands

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