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Assaf Zeevi*

Graduate School of Business

Columbia University

*Based on joint work with Denis Saure

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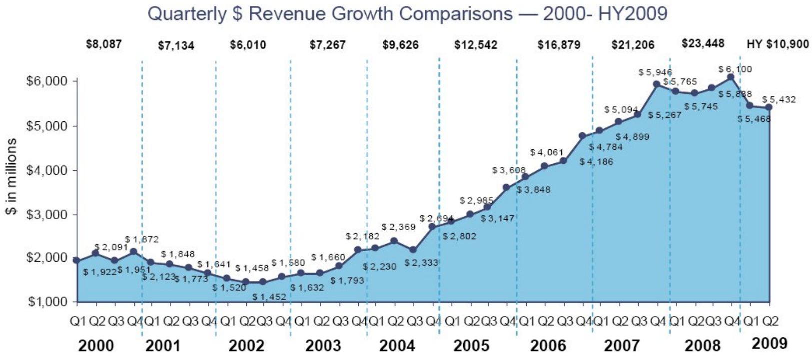

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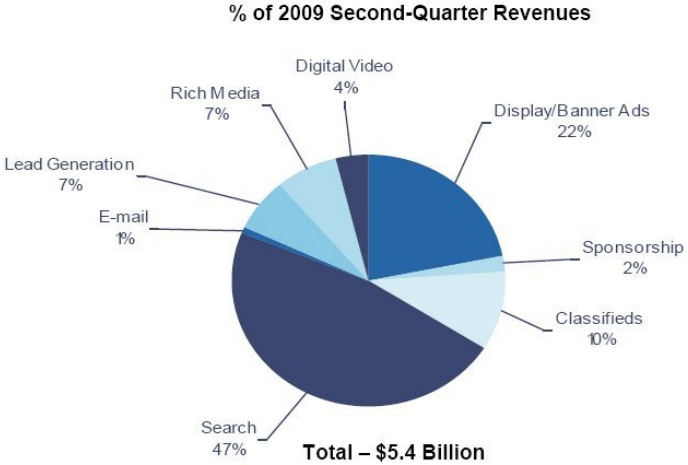

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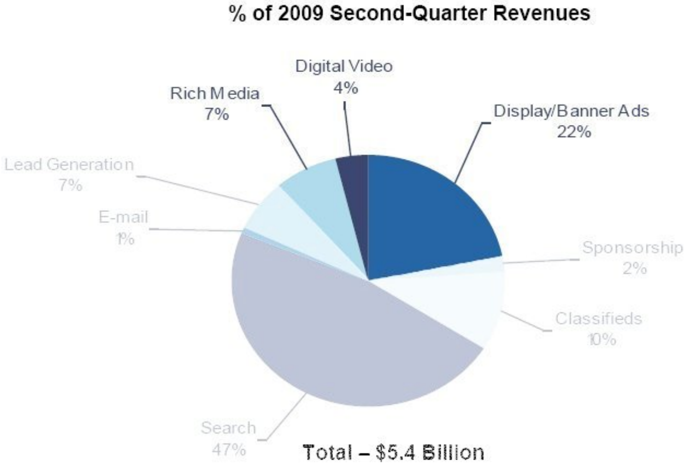

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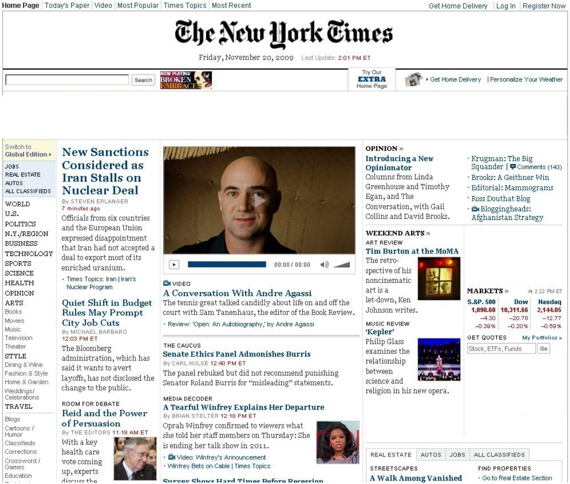

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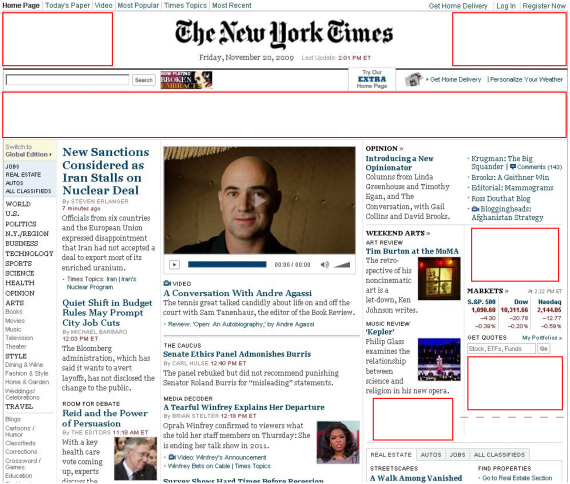

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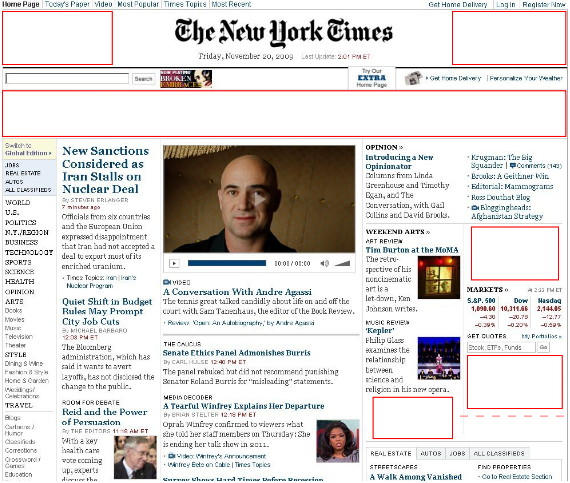

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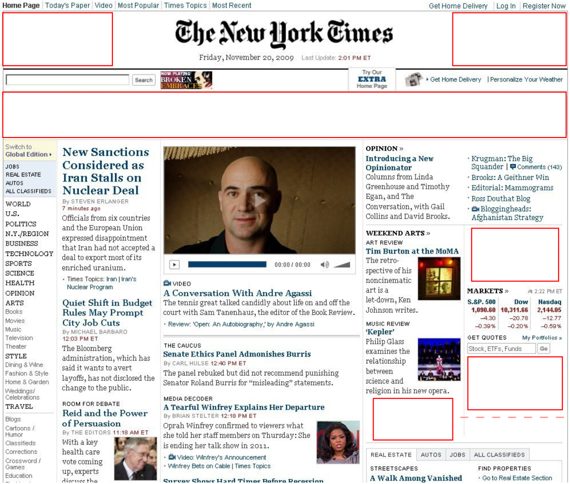

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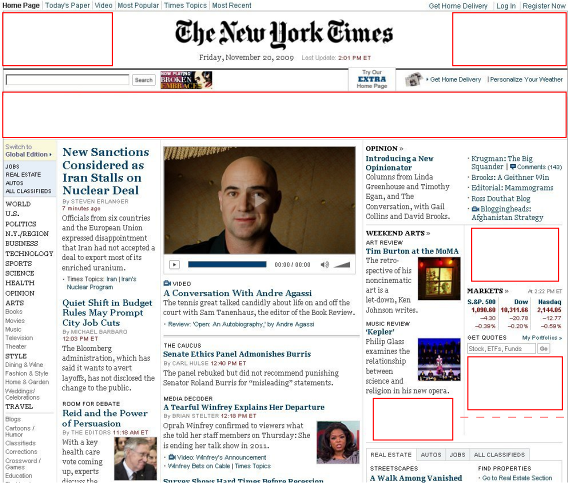

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The retro-spective of his noncinematic art is a let-down, Ken Johnson writes.

MUSIC REVIEW
'Kepler'
Philip Glass examines the relationship between science and religion in his new opera.

MARKETS » At 2:22 PM ET

S.&P. 500	Dow	Nasdaq
1,090.60	10,311.66	2,144.05
−4.30	−20.78	−12.77
−0.39%	−0.20%	−0.59%

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Ad-mix

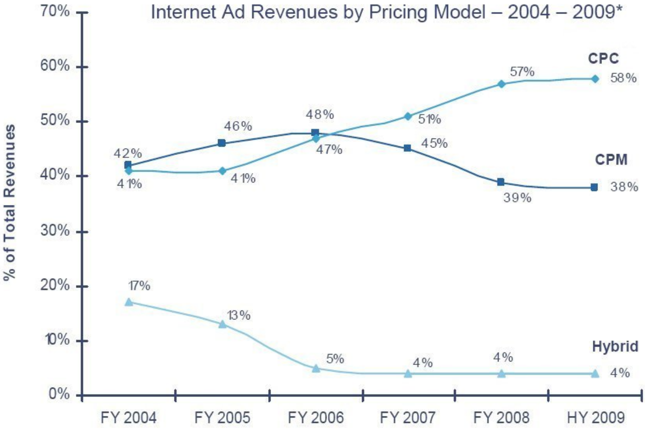
Profit maximization

- ad pool
- user information
- ad/user performance

pricing model. . .

1. cost per mille (CPM)
2. cost per click (CPC)

Online Advertisement: Pricing Models



source: Interactive Advertisement Bureau Internet Advertisement Revenue Report (by PricewaterhouseCoopers)

Customization in Online Advertisement

- anonymity on the Internet

"On the Internet, nobody knows you're a dog."

July 1993, *The New Yorker*

Customization in Online Advertisement

- anonymity on the Internet
- common practice
  - internet cookies
  - list of categories of interest
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id=c51781920000060||t=1253117509|et=730
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- user information
  - behavioral, geographical, demographical data ...

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Online Advertisement: Salient Features

Advent of the Internet has transformed consumer experience of advertisements and media. . .

dynamic/customized advertisement display

- one-to-one interaction with users. . .
- contracts (CPC) are increasingly performance-based
- customization to individual users exploiting side information
- dynamic decision making to balance learning and profits

Road Map

Focus: Publisher's display decision in dynamic environment

I. Customization in online advertisement
   - publisher's problem definition
   - need for *dynamic learning* of ad performance

II. Stylized model for display-based online advertisement
   - limit of achievable performance
   - policy construction and guarantees

III. Insights and takeaway messages

Towards a Problem Formulation...

Publisher's decision: ad/user performance

1. direct revenue: cost per click (cpc)
2. click probability:

$$\text{user profile} + \text{ad mix} \rightarrow \text{click probability}$$

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model parameters

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- additional considerations: display capacity...

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Publisher's objective: ideally...

maximize expected revenue from interaction with users

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estimation accuracy vs profit maximization

Related Literature

Learning approach to interactive marketing

- Gooley and Lattin (2000)
  - message customization
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Multi Armed Bandit (MAB) Literature

- Slivkins (2009), Lu et al (2009)
  - side information: MAB in metric spaces

Roadmap

I. Customization in online advertisement

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Problem Formulation

Stylized model for display-based online advertisement

- finite users (T) arrive sequentially
- finite pool of ads ($\mathcal{N}$) with given profit margins (w_i)
- ad-mix ($s \in \mathcal{S}$)...

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- **objective**: maximize revenue by suitable ad display policy
- user clicks on at most one ad ...
- users are utility maximizers

U(user profile, ad) + ad-mix $\rightarrow$ click decision

Problem Formulation: User Utilities

Logit model with user-specific mean utility

utility of ad i noise (Gumbel)

$$U_i = \beta_i \cdot x + \epsilon_i$$

ad factors user profile

- user profile x is d-dimensional vector [observed]
- ad factors β_i is d-dimensional vector [to be estimated]

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$$\begin{array}{cl} x & \\ \begin{pmatrix} 0.4 \\ 0.9 \\ 35 \\ 1 \end{pmatrix} & \begin{array}{l} \text{sport affinity} \\ \text{prob. male} \\ \text{exp. age} \\ \text{dummy} \end{array} \end{array}$$

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our approach: Logistic regression (profiles)

$$f_i(\underbrace{s}_{\text{ad mix}}, x, \beta) = \frac{\exp\{\beta_i \cdot x\}}{1 + \sum_{j \in s} \exp\{\beta_j \cdot x\}}$$

Problem Formulation

- expected revenue from displaying ad mix s to user profile x:

$$r(s, x, \beta) = \sum_{i \in s} w_i \cdot \left(\frac{\exp\{\beta_i \cdot x\}}{1 + \sum_{j \in s} \exp\{\beta_j \cdot x\}} \right)$$

ad profit margin

logit click prob.

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ad i factors β_i initially unknown for all ads

Oracle Benchmark

Suppose publisher knows β a priori

- formulate and solve an optimization problem

$$J^*(T|\beta) := \sup_{s(\cdot)} \mathbb{E}\left[\sum_{t=1}^{T} r(s(t), X_t, \beta)\right]$$

known parameters

feasible ad policies

expected revenue

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Oracle policy: offer $s^*(X_t, \beta)$ to user t

$$s^*(x, \beta) \in \operatorname{argmax}\{r(s, x, \beta) \,:\, s \in \mathcal{S}\}$$

expected revenue

Measuring Policy Performance

- ad mix decision for feasible policies based on history of past interaction and current user profile

- performance of ad mix policy π:

  revenue loss relative to oracle policy

$$\mathcal{R}(\pi, T) := J^*(T|\beta) - \mathbb{E}\left[\sum_{t=1}^{T} r(s^\pi(t), X_t, \beta)\right]$$

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expected revenue

Main Q: **how small** can we make this revenue loss?

structure of an **optimal policy?**

Limit of Achievable Performance

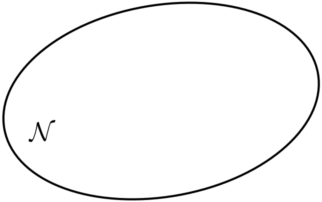
Theorem [Saure and Z (2012)]

Any *good* policy π must incur revenue loss

$$\mathcal{R}(\pi, T) \geq \sum_{i \in \mathcal{N}} K_i \log T$$

Limit of Achievable Performance

- Fix user profile $x \in \mathcal{X}$:



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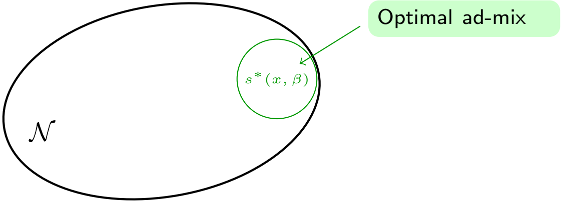

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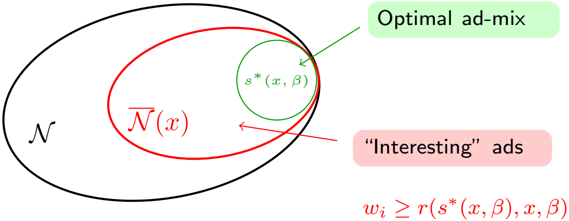

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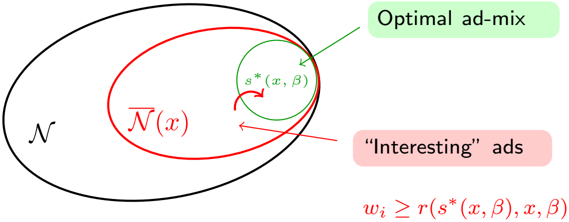

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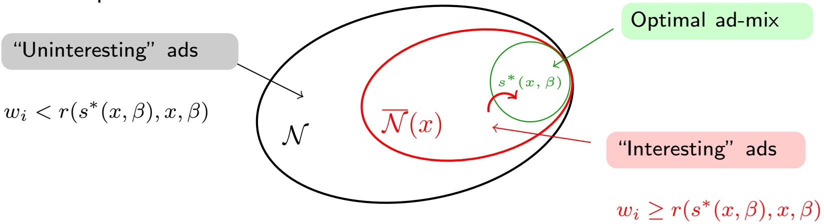

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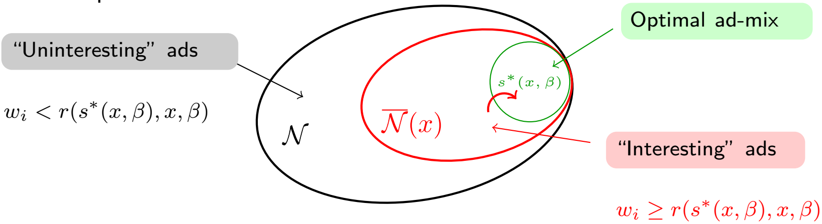

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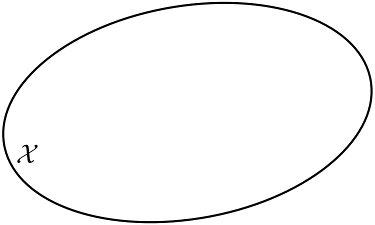

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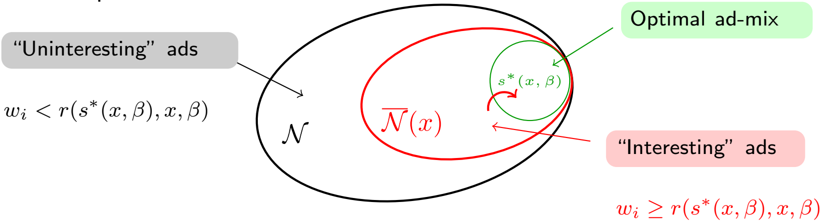

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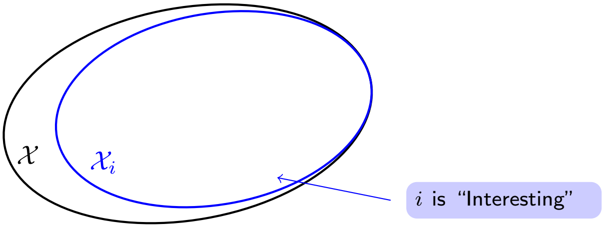

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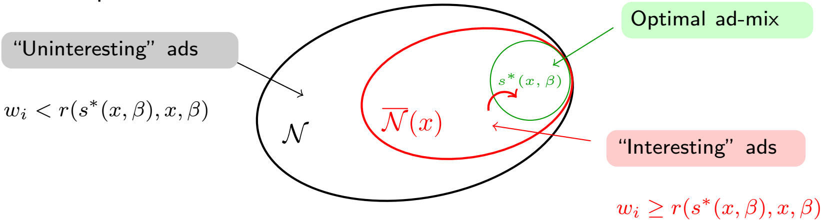

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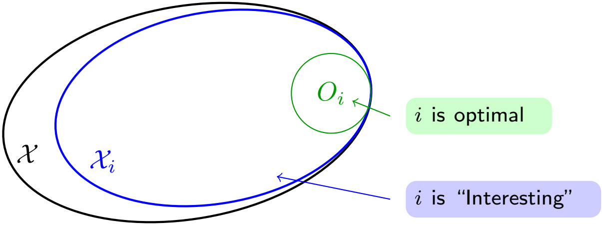

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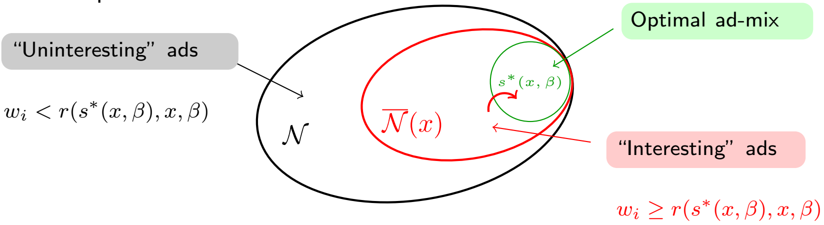

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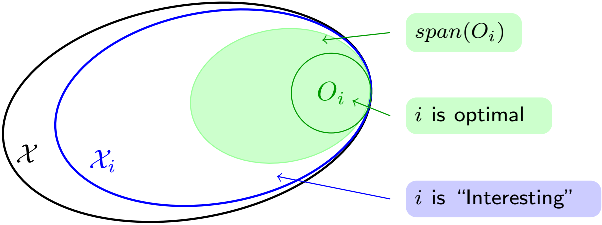

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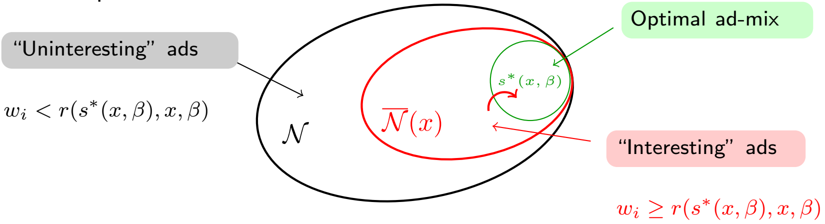

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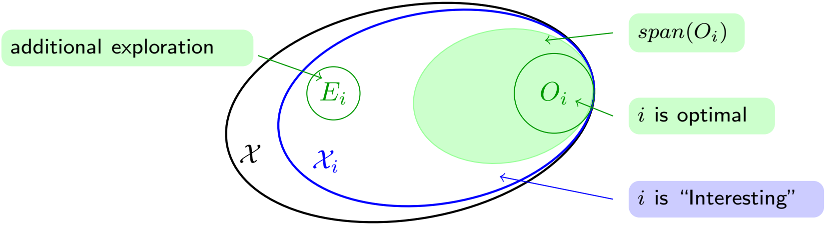

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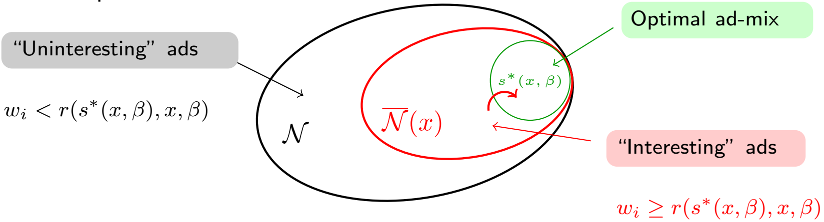

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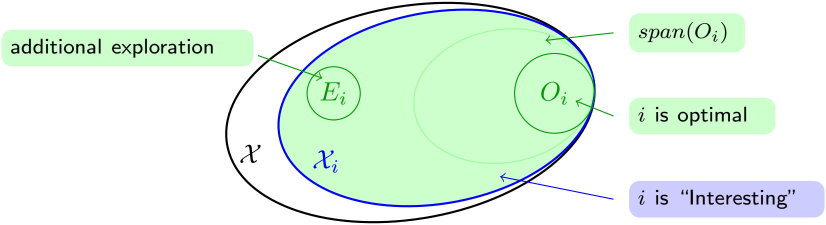

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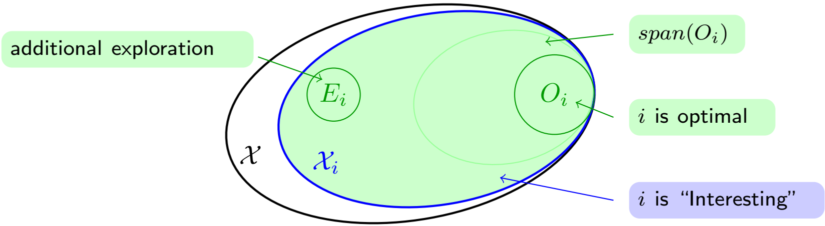

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$K_i \sim rank(\mathcal{X}_i) - rank(O_i)$

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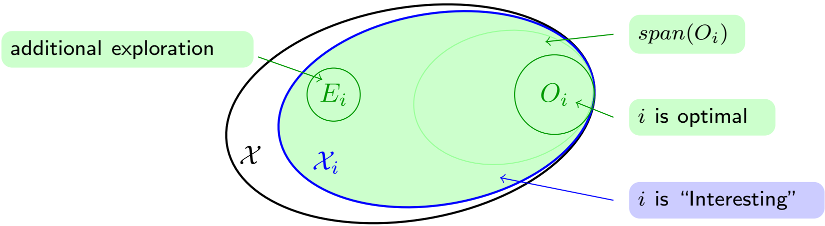

Qualitative Insights and Policy Design

Ad/profile exploration as source of revenue loss

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- for a given ad, there is no need to estimate mean utilities for every profile
  - need to assess performance only on some profiles ($\mathcal{X}_i$)
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  - need to assess performance only on some profiles ($\mathcal{X}_i$)
  - use information on set spanning such profiles
- use information that does not contribute to revenue loss
  - use profiles for which an ad is optimal
- information contributing to revenue loss must be capped
  - performed on order $\log T$ users...

Policy design: Key ingredients

Intuition: force right frequency of ad i experimentation on suitable estimation-set ($E_i \in \mathcal{X}$)

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$$\widehat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\#\text{ clicks on ad } i \text{ for profile } x}{\#\text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x)\,,\ x \in E_i \right\}$$

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- adapt E_i to span a proxy for $\mathcal{X}_i \ldots$

Policy design: Key ingredients

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  - otherwise, EXPLOIT approximate oracle solution $s^*(X_t, \widehat{\beta})$

Performance of the Proposed Policy

Theorem [Saure and Z (2012)]

For suitable chosen tuning parameter κ,

$$\mathcal{R}(\pi^*, T) \leq \overline{K} \sum_{i \in \mathcal{N}} (rank(\mathcal{X}_i) - rank(O_i)) \log T + K,$$

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Key results: for each profile

- *uninteresting ads* displayed to finite (independent of T) number of users
- *ads in the optimal mix* displayed outside that mix finitely many times

Proof Sketch

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min optimality gap across profiles	$+$	continuity of expected revenue w.r.t β	$\Rightarrow$	threshold on estimation error

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- balance exploration and exploitation error $(\kappa > c^{-1})$

$$\mathcal{R}(\pi^*, T) \leq O\left(\kappa \log T + \sum_{t=1}^{T} \frac{1}{t^{c\kappa}}\right)$$

Numerical Illustration

- 4 products, 3 two-dimensional profiles
- feasible set $\mathcal{S} := \{s \subset \mathcal{N} \,:\, |s| \leq 2\}$, $\kappa = 40$

$$\beta = \begin{pmatrix} -1.30 & 2.00 & 2.75 & 3.00 \\ 3.00 & 2.00 & 2.75 & -1.30 \end{pmatrix} \qquad \mathcal{X} = \begin{pmatrix} 0.1 & 0.5 & 0.9 \\ 0.9 & 0.5 & 0.1 \end{pmatrix}$$

Oracle solution

profile	x_1	x_2	x_3
opt. mix	$\{1,2\}$	$\{2,3\}$	$\{2,4\}$
opt. revenue	0.587	0.546	0.578
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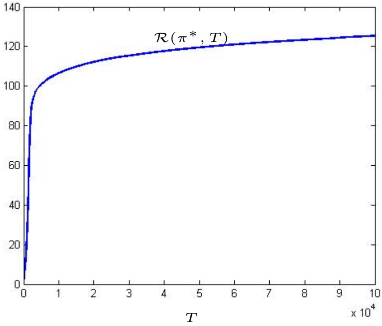

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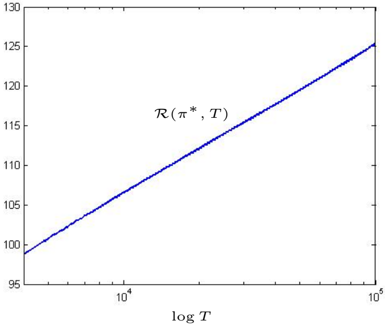

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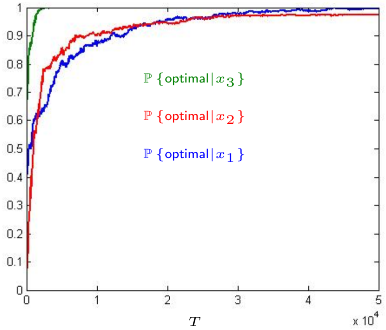

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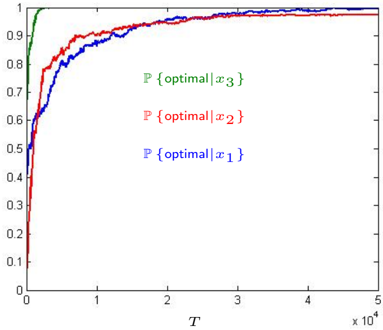
anon. mix	$\{1,2\}$	$\{1,2\}$	$\{1,2\}$
anon. revenue	0.587	0.543	0.525



Roadmap

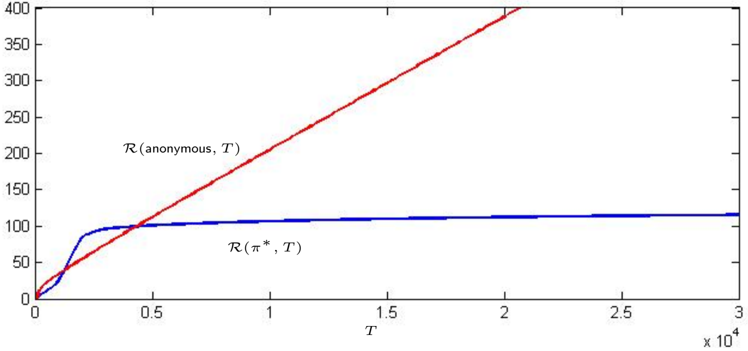
I. Customization in online advertisement

II. Stylized model for display-based online advertisement

III. Insights and takeaway messages

Insights and Takeaway Messages

- value of customization
  - speed of learning
  - misspecification risk



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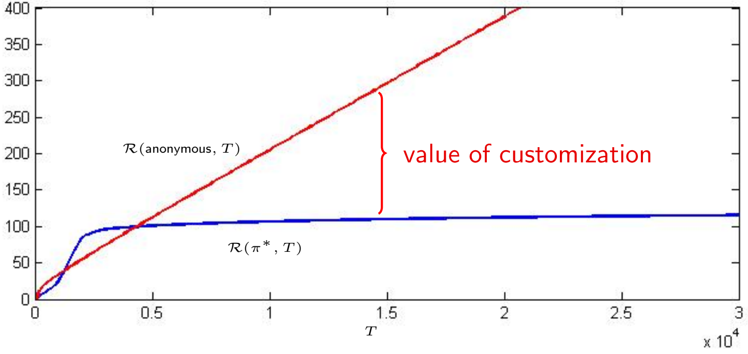

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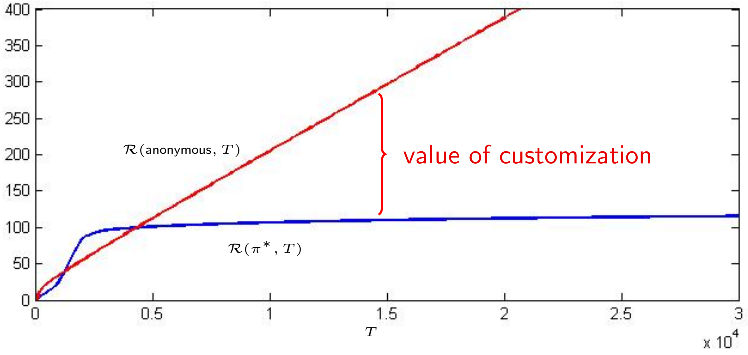

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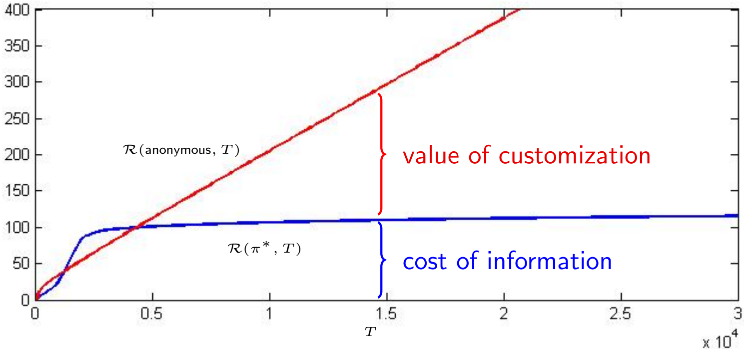

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- related recent applications of MAB
  - dynamic content referral [Besbes, Gur and Z (2012a)]
  - temperature tracking and restless bandits [Besbes and Z (2012b)]
  - personalization (Pandora, various recommendation systems etc)
  - dynamic design of experiments / screening
  - cognitive radio [Lai et al (2011)]
  - mechanism design formulation [Kakade et al (2012)]