

# MULTI-ARMED BANDITS: APPLICATIONS TO ONLINE ADVERTISING

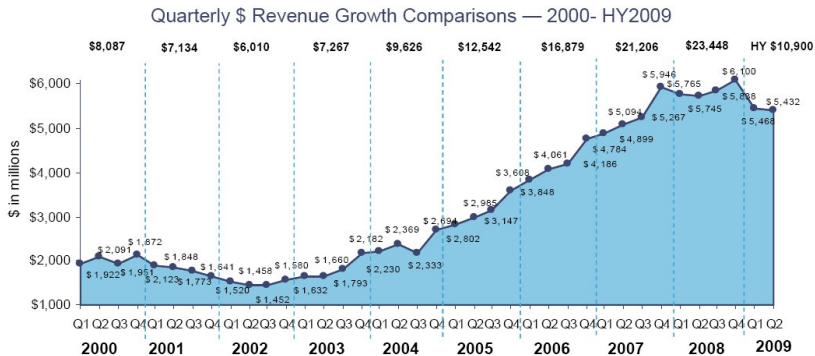
---

Assaf Zeevi\*

Graduate School of Business  
Columbia University

\*Based on joint work with Denis Saure

# Online Advertisement: Industry Overview

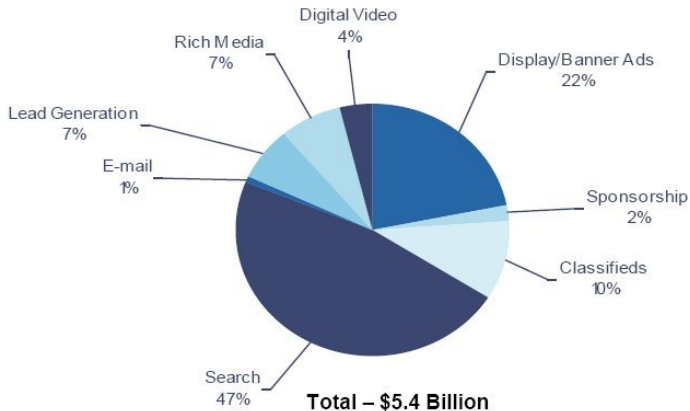


source: Interactive Advertisement Bureau Internet Advertisement Revenue Report (by PricewaterhouseCoopers)

# Online Advertisement: Industry Overview

---

% of 2009 Second-Quarter Revenues

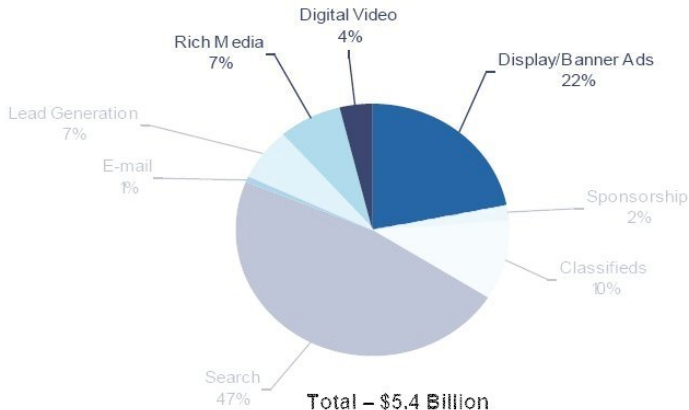


source: Interactive Advertisement Bureau Internet Advertisement Revenue Report (by PricewaterhouseCoopers)

# Online Advertisement: Industry Overview

---

% of 2009 Second-Quarter Revenues



source: Interactive Advertisement Bureau Internet Advertisement Revenue Report (by PricewaterhouseCoopers)

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

## The New York Times

Friday, November 20, 2009 Last Update: 2:01 PM ET

Search



Try Our  
EXTRA  
Home Page



Get Home Delivery | Personalize Your Viesher

Switch to  
Global Edition +

- JOBS
- REAL ESTATE
- AUTOS
- ALL CLASSIFIEDS
- WORLD
- U.S.
- POLITICS
- N.Y./REGION
- BUSINESS
- TECHNOLOGY
- SPORTS
- SCIENCE
- HEALTH
- OPINION
- ARTS
- Books
- Movies
- Music
- Television
- Theater
- STYLE
- Dining & Wine
- Fashion & Style
- Home & Garden
- Weddings/
- Celebrations
- TRAVEL
- Blog
- Cartoons /
- Humor
- Classifieds
- Connections
- Crossword /
- Games
- Education

### New Sanctions Considered as Iran Stalls on Nuclear Deal

By STEVEN ERLANGER  
7 minutes ago

Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.

Times Topics: Iran | Iran's Nuclear Program

### Quiet Shift in Budget Rules May Prompt City Job Cuts

By MICHAEL BARBARO  
12:02 PM ET

The Bloomberg administration, which has said it wants to avert layoffs, has not disclosed the change to the public.

### ROOM FOR DEBATE Reid and the Power of Persuasion

By THE EDITORS 11:18 AM ET

With a key health care vote coming up, experts discuss the



VIDEO

### A Conversation With Andre Agassi

The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.

Review: 'Open: An Autobiography,' by Andre Agassi

THE CAUCUS

### Senate Ethics Panel Admonishes Burr

By GREG ROUSE 12:40 PM ET

The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

MEDIA DECODER

### A Tearful Winfrey Explains Her Departure

By BRIAN STELTER 12:18 PM ET

Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.

Video: Winfrey's Announcement  
Winfrey Bets on Cable | Times Topics



Viewers Show Hard Times Before Departure

OPINION +

### Introducing a New Opinionator

Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gill Collins and David Brooks.

- Krugman: The Big Squads' | Comments (147)
- Brooks: A Gethner Win
- Editorial: Mammograms
- Ross Douthat Blog
- Bloggingheads: Afghanistan Strategy

WEEKEND ARTS +

ART REVIEW

### Tim Burton at the MoMA

The retrospective of his noncinematic art is a let-down, Ken Johnson writes.



MUSIC REVIEW

### 'Kepler'

Philip Glass examines the relationship between science and religion in his new opera.



MARKETS +

At 2:22 PM ET

S&P 500	Dow	Nasdaq
1,899.58	18,311.65	2,164.05
-4.30	+20.75	+12.77
-0.30%	+0.20%	+0.60%

GET QUOTES

My Portfolio +

Stock, ETFs, Funds | Go

REAL ESTATE | AUTOS | JOBS | ALL CLASSIFIEDS

STREETSCAPES  
A Walk Among Vanished

FIND PROPERTIES  
Go to Real Estate Section

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

# The New York Times

Friday, November 26, 2009 Last Update: 2:01 PM ET

Search  Try Our EXTRA Home Page | Get Home Delivery | Personalize Your Viesher

Switch to Global Edition •

- JOBS
- REAL ESTATE
- AUTOS
- ALL CLASSIFIEDS
- WORLD
- U.S.
- POLITICS
- N.Y./REGION
- BUSINESS
- TECHNOLOGY
- SPORTS
- SCIENCE
- HEALTH
- OPINION
- ARTS
- Books
- Movies
- Music
- Television
- Theater
- STYLE
- Dining & Wine
- Fashion & Style
- Home & Garden
- Weddings/ Celebrations
- TRAVEL
- Blog
- Cartoons / Humor
- Classifieds
- Connections
- Crossword / Games
- Education

## New Sanctions Considered as Iran Stalls on Nuclear Deal

By STEVEN ERLANGER  
7 minutes ago

Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.

Times Topics: Iran / Iran's Nuclear Program

**Quiet Shift in Budget Rules May Prompt City Job Cuts**


By MICHAEL BARBARO  
12:02 PM ET

The Bloomberg administration, which bar said it wants to avert layoffs, has not disclosed the change to the public.

**ROOM FOR DEBATE Reid and the Power of Persuasion**

By THE EDITORS  
11:18 AM ET

With a key health care vote coming up, experts discuss the



VIDEO  
**A Conversation With Andre Agassi**  
The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.

Review: 'Open: An Autobiography,' by Andre Agassi

**THE CAUCUS Senate Ethics Panel Admonishes Burr**

By GREG HOLZE  
12:40 PM ET

The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

**MEDIA DECODER A Tearful Winfrey Explains Her Departure**

By BRIAN STELTER  
12:18 PM ET

Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.

Video: Winfrey's Announcement  
Winfrey Bets on Cable | Times Topics

**VIDEO Show Used Times Before Departure**

**OPINION**

**Introducing a New Opinionator**  
Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gill Collins and David Brooks.

Krugman: The Big Squads' | Comments (940)  
Brooks: A Gethner Win  
Editorial: Mammograms  
Ross Douthat Blog  
Bloggingheads: Afghanistan Strategy

**WEEKEND ARTS**

**ART REVIEW Tim Burton at the MoMA**  
The retrospective of his noncinematic art is a let-down, Ken Johnson writes.

**MUSIC REVIEW 'Kepler'**  
Philip Glass examines the relationship between science and religion in his new opera.

**MARKETS** | 11:22 PM ET

S&P 500	Dow	Nasdaq
1,899.58	18,311.65	2,164.05
+4.50	+20.75	+12.77
+0.30%	+0.20%	+0.60%

**GET QUOTES** | My Portfolio •  
Stock, ETFs, Funds | Go

REAL ESTATE | AUTOS | JOBS | ALL CLASSIFIEDS  
STREETSCAPES | FIND PROPERTIES  
**A Walk Among Vanished** | Go to Real Estate Section

Ad-mix

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

# The New York Times

Friday, November 26, 2009 Last Update: 2:01 PM ET

Search [input] [button] Try Our EXTRA Home Page [button] Get Home Delivery | Personalize Your Viewer [button]

Switch to Global Edition •

- JOBS
- REAL ESTATE
- AUTOS
- ALL CLASSIFIED
- WORLD
- U.S.
- POLITICS
- N.Y./REGION
- BUSINESS
- TECHNOLOGY
- SPORTS
- SCIENCE
- HEALTH
- OPINION
- ARTS
- Books
- Movies
- Music
- Television
- Theater
- STYLE
- Dining & Wine
- Fashion & Style
- Home & Garden
- Weddings/ Celebrations
- TRAVEL
- Blog
- Cartoons / Humor
- Classifieds
- Connections
- Crossword / Games
- Education

## New Sanctions Considered as Iran Stalls on Nuclear Deal

By STEVEN ERLANGER  
7 minutes ago

Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.

Times Topics: Iran / Iran's Nuclear Program

**Quiet Shift in Budget Rules May Prompt City Job Cuts**

By MICHAEL BARBARO  
12:02 PM ET

The Bloomberg administration, which has said it wants to avert layoffs, has not disclosed the change to the public.

**ROOM FOR DEBATE**  
**Reid and the Power of Persuasion**

By THE EDITORS  
11:18 AM ET

With a key health care vote coming up, experts discuss the



**VIDEO**  
**A Conversation With Andre Agassi**  
The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.

Review: 'Open: An Autobiography,' by Andre Agassi

**THE CAUCUS**  
**Senate Ethics Panel Admonishes Burr**

By GREG HOLDS  
12:40 PM ET

The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

**MEDIA DECODER**  
**A Tearful Winfrey Explains Her Departure**

By BRIAN STELTER  
12:18 PM ET

Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.

Video: Winfrey's Announcement  
Winfrey Bets on Cable | Times Topics

**OPINION**

**Introducing a New Opinionator**  
Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gill Collins and David Brooks.

Krugman: The Big Squads? | Comments (140)  
Brooks: A Gethner Win  
Editorial: Mammograms  
Ross Douthat Blog  
Bloggingheads: Afghanistan Strategy

**WEEKEND ARTS**

**ART REVIEW**  
**Tim Burton at the MoMA**

The retrospective of his noncinematic art is a let-down, Ken Johnson writes.

**MUSIC REVIEW**  
**'Kepler'**

Philip Glass examines the relationship between science and religion in his new opera.

**MARKETS** 11/23 PM ET

	S&P 500	Dow	Nasdaq
Change	+4.50	+20.75	+12.77
% Change	+0.30%	+0.30%	+0.60%

**GET QUOTES** My Portfolio >  
Stock, ETFs, Funds | Go

REAL ESTATE | AUTOS | JOBS | ALL CLASSIFIED

**STREETSCAPES** | **FIND PROPERTIES**

**A Walk Among Vanished** | **Go to Real Estate Section**

Ad-mix

Profit maximization

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

# The New York Times

Friday, November 26, 2009 Last Update: 2:01 PM ET

Search [input] [button] Try Our EXTRA Home Page [button] Get Home Delivery | Personalize Your Viewer [button]

Switch to Global Edition •

- JOBS
- REAL ESTATE
- AUTOS
- ALL CLASSIFIEDS
- WORLD
- U.S.
- POLITICS
- N.Y./REGION
- BUSINESS
- TECHNOLOGY
- SPORTS
- SCIENCE
- HEALTH
- OPINION
- ARTS
- Books
- Movies
- Music
- Television
- Theater
- STYLE
- Dining & Wine
- Fashion & Style
- Home & Garden
- Weddings/ Celebrations
- TRAVEL
- Blog
- Cartoons / Humor
- Classifieds
- Connections
- Crossword / Games
- Education

## New Sanctions Considered as Iran Stalls on Nuclear Deal

By STEVEN ERLANGER  
7 minutes ago

Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.

Times Topics: Iran | Iran's Nuclear Program

**Quiet Shift in Budget Rules May Prompt City Job Cuts**

By MICHAEL BARBARO  
12:02 PM ET


The Bloomberg administration, which has said it wants to avert layoffs, has not disclosed the change to the public.

**ROOM FOR DEBATE**

### Reid and the Power of Persuasion

By THE EDITORS  
11:18 AM ET

With a key health care vote coming up, experts discuss the



VIDEO  
**A Conversation With Andre Agassi**  
The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.

Review: 'Open: An Autobiography,' by Andre Agassi

### Senators Ethics Panel Admonishes Burr

By GREG ROUSE  
12:40 PM ET

The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

**MEDIA DECODER**

### A Tearful Winfrey Explains Her Departure

By BRIAN STELTER  
12:18 PM ET

Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.

Video: Winfrey's Announcement  
Winfrey Bets on Cable | Times Topics

**VIDEO SHOW UP! TIME TO DANCE**

### OPINION

**Introducing a New Opinionator**  
Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gill Collins and David Brooks.

Krugman: The Big Squads? | Comments (942)  
Brooks: A Gethner Win  
Editorial: Mammograms  
Ross Douthat Blog  
Bloggingheads: Afghanistan Strategy

### WEEKEND ARTS

**ART REVIEW**  
**Tim Burton at the MoMA**  
The retrospective of his noncinematic art is a let-down, Ken Johnson writes.

**MUSIC REVIEW**  
**'Kepler'**  
Philip Glass examines the relationship between science and religion in his new opera.

### MARKETS

As of 2:22 PM ET

	S&P 500	Dow	Nasdaq
Change	+4.50	+20.75	+12.77
% Change	+0.30%	+0.30%	+0.65%

**GET QUOTES** My Portfolio >  
Stock, ETFs, Funds | Go

REAL ESTATE | AUTOS | JOBS | ALL CLASSIFIEDS

**STREETSCAPES** | **FIND PROPERTIES**

**A Walk Among Vanished** | **Go to Real Estate Section**

Ad-mix

Profit maximization

■ ad pool



# Customization in Online Advertisement

The screenshot shows the homepage of The New York Times. At the top, there are navigation links for Home Page, Today's Paper, Video, Most Popular, Times Topics, and Most Recent. The main masthead features the newspaper's name, the date (Friday, November 26, 2009), and the time (Last Update: 2:01 PM ET). Below the masthead is a search bar and a section for 'Try Our EXTRA Home Page'. The main content area is divided into several columns. On the left is a vertical navigation menu with categories like JOBS, REAL ESTATE, AUTOS, and WORLD. The first main article is 'New Sanctions Considered as Iran Stalls on Nuclear Deal' by Steven Erlanger, featuring a video player for 'A Conversation With Andre Agassi'. Other articles include 'Quiet Shift in Budget Rules May Prompt City Job Cuts' and 'Senators Ethics Panel Admonishes Burr'. On the right side, there are sections for 'OPINION' (Introducing a New Opiniator), 'WEEKEND ARTS' (Art Review: Tim Burton at the MoMA), and 'MARKETS'. At the bottom, there are sections for 'REAL ESTATE', 'STREETSCAPES', and 'A Walk Among Vanished'. A red dashed box highlights several areas: the masthead, the search bar, the video player, and several advertisement spots (represented by empty boxes) located in the top right, middle right, and bottom right of the page.

Ad-mix

Profit maximization

- ad pool
- user information

# Customization in Online Advertisement

The screenshot shows the homepage of The New York Times. At the top, there are navigation links for Home Page, Today's Paper, Video, Most Popular, Times Topics, and Most Recent. On the right, there are links for Get Home Delivery, Log In, and Register Now. The main header features the newspaper's name, "The New York Times", and the date, "Friday, November 26, 2009", with a last update time of 2:01 PM ET. Below the header is a search bar and a "Try Our EXTRA Home Page" button. The main content area is divided into several sections: "New Sanctions Considered as Iran Stalls on Nuclear Deal" by Steven Erlanger, "A Conversation With Andre Agassi" by Sam Tanenhaus, "Quiet Shift in Budget Rules May Prompt City Job Cuts" by Michael Barbaro, "Senators Ethics Panel Admonishes Burr" by Carol Hulse, and "A Tearful Winfrey Explains Her Departure" by Brian Stelter. There are also sections for "WEEKEND ARTS" and "MARKETS". On the left side, there is a "Switch to Global Edition" button and a list of categories including Jobs, Real Estate, Autos, and more. On the right side, there are "OPINION" and "WEEKEND ARTS" sections, a "MARKETS" table, and "OET QUOTES".

Ad-mix

Profit maximization

- ad pool
- user information
- ad/user performance

# Customization in Online Advertisement

The screenshot shows the homepage of The New York Times. At the top, there are navigation links for Home Page, Today's Paper, Video, Most Popular, Times Topics, and Most Recent. On the right, there are links for Get Home Delivery, Log In, and Register Now. The main header features the newspaper's name, the date (Friday, November 26, 2009), and the time (Last Update: 2:01 PM ET). Below the header is a search bar and a section for 'Try Our EXTRA Home Page' with options for Get Home Delivery and Personalize Your Viesher.

The main content area is divided into several columns:

- Left Column:** A vertical menu with categories like Global Edition, Jobs, Real Estate, Autos, All Classifieds, World, U.S., Politics, N.Y./Region, Business, Technology, Sports, Science, Health, Opinion, Arts, Books, Movies, Music, Television, Theater, Style, Dining & Wine, Fashion & Style, Home & Garden, Weddings/Celebrations, Travel, Blogs, Cartoons, Humor, Classifieds, Corrections, Crossword/Games, and Education.
- Second Column:** A featured article titled "New Sanctions Considered as Iran Stalls on Nuclear Deal" by Steven Erlanger, 7 minutes ago. It includes a video player for "A Conversation With Andre Agassi" and a "The Caucus" section titled "Senate Ethics Panel Admonishes Burr".
- Third Column:** An "Opinion" section titled "Introducing a New Opinionator" by Linda Greenhouse and Timothy Egan, and a "Weekend Arts" section titled "Tim Burton at the MoMA".
- Right Column:** A "Markets" section showing stock prices for S&P 500, Dow, and Nasdaq, and a "Get Quotes" section for Stocks, ETFs, and Funds.

Red dashed boxes and arrows highlight specific areas: one box around the top navigation and header, another around the search bar and 'Try Our EXTRA' section, and a large box on the right side of the page containing the text 'Ad-mix', 'Profit maximization', and a list of items: 'ad pool', 'user information', and 'ad/user performance'. A red arrow points from the 'Ad-mix' text to the 'Profit maximization' text. Below this list, the text 'pricing model...' is also present.

Ad-mix

Profit maximization

- ad pool
- user information
- ad/user performance

pricing model...

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

**The New York Times**  
Friday, November 20, 2009 Last Update: 2:01 PM ET

TAKE YOUR OWN PATH

SEARCH [input] [button] Try Our EXTRA Home Page [button] Get Home Delivery | Personalize Your Website [button]

**BUSINESS** I TAKE MY OWN PATH. I CHOOSE DELL. LEARN MORE

Switch to Global Edition •

**JOBS** REAL ESTATE AUTOS ALL CLASSIFIED

**WORLD** U.S. POLITICS N.Y./REGION BUSINESS TECHNOLOGY SPORTS SCIENCE HEALTH OPINION ARTS Books Movies Music Television Theater STYLE Dining & Wine Fashion & Style Home & Garden Weddings/Celebrations TRAVEL Blogs Cartoons / Humor Classifieds Comixes Crossword / Games Education

**New Sanctions Considered as Iran Stalls on Nuclear Deal**  
By STEVEN ERLANGER 7 minutes ago  
Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.  
Times Topics: Iran / Iran's Nuclear Program

**Quiet Shift in Budget Rules May Prompt City Job Cuts**  
By MICHAEL BARBARO 12:50 PM ET  
The Bloomberg administration, which has said it wants to avert layoffs, has not disclosed the change to the public.

**ROOM FOR DEBATE Reid and the Power of Persuasion**  
By THE EDITORS 11:18 AM ET  
With a key health care vote coming up, experts discuss the

**OPINION** Introducing a New Opinionator  
Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gail Collins and David Brooks.

**WEEKEND ARTS** ART REVIEW  
**Tim Burton at the MoMA**  
The retrospective of his noncinematic art is a let-down, Ken Johnson writes.

**MUSIC REVIEW** "Kepler"  
Philip Glass examines the relationship between science and religion in his new opera.

**FALL OPEN HOUSE**  
HINSHIA UNIVERSITY NOVEMBER 21

**REAL ESTATE** AUTOS JOBS ALL CLASSIFIED  
**STREETSCAPES** FIND PROPERTIES  
**A Walk Among Vanished** - Go to Real Estate Section

**MARKETS** 11:22 PM ET  
S&P 500 Dow Nasdaq  
1,899.48 18,311.66 2,164.05  
-4.50 -20.7% +12.77  
-0.30% -0.20% -0.65%

**GET QUOTES** My Portfolio  
Stock, ETFs, Funds [Go]

**THIS HOLIDAY SEASON RECOVER THE JOY OF FAMILY**  
ROBERT DE NIRO  
**EVERYBODY'S FINE**  
INTERVIEW FOR SALE

**VIDEO**  
**A Conversation With Andre Agassi**  
The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.  
- Review: "Open: An Autobiography," by Andre Agassi

**THE CAUCUS**  
**Senate Ethics Panel Admonishes Burr**  
The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

**MEDIA DECODER**  
**A Tearful Winfrey Explains Her Departure**  
By BRIAN STELTER 12:18 PM ET  
Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.  
Video: Winfrey's Announcement  
- Winfrey Bets on Cable | Times Topics

**VIDEO**  
**Steve Jobs Had Time Before Departure**

**OPINION** TAKE MY OWN PATH. I CHOOSE DELL. LEARN MORE

**WEEKEND ARTS** ENTER THE 1ST VIRTUAL MASTO STORE

**MARKETS** S&P 500 Dow Nasdaq

**GET QUOTES** My Portfolio

**THIS HOLIDAY SEASON RECOVER THE JOY OF FAMILY**

**EVERYBODY'S FINE**

Ad-mix

Profit maximization

- ad pool
- user information
- ad/user performance

pricing model...

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

**The New York Times**  
Friday, November 20, 2009 Last Update: 2:01 PM ET

TAKE YOUR OWN PATH

SEARCH [input] [button] Try Our EXTRA Home Page [button] Get Home Delivery | Personalize Your Viesites [button]

BUSINESS **I TAKE MY OWN PATH. I CHOOSE DELL.** LEARN MORE [button]

Switch to Global Edition •

**JOBS**  
REAL ESTATE  
AUTOS  
ALL CLASSIFIEDS

**WORLD**  
U.S.  
POLITICS  
N.Y./REGION  
BUSINESS  
TECHNOLOGY  
SPORTS  
SCIENCE  
HEALTH  
OPINION  
ARTS  
Books  
Movies  
Music  
Television  
Theater  
STYLE  
Dining & Wine  
Fashion & Style  
Home & Garden  
Weddings/  
Celebrations  
TRAVEL

**Blog**  
Cartoons /  
Humor  
Classifieds  
Columns  
Crossword /  
Games  
Education

**New Sanctions Considered as Iran Stalls on Nuclear Deal**  
By STEVEN ERLANGER  
7 minutes ago  
Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.  
Times Topics: Iran / Iran's Nuclear Program

**Quiet Shift in Budget Rules May Prompt City Job Cuts**  
By MICHAEL BARBARO  
12:02 PM ET  
The Bloomberg administration, which has said it wants to avert layoffs, has not disclosed the change to the public.

**ROOM FOR DEBATE Reid and the Power of Persuasion**  
By THE EDITORS  
11:10 AM ET  
With a key health care vote coming up, experts discuss the

**VIDEO**  
**A Conversation With Andre Agassi**  
The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.  
Review: 'Open: An Autobiography,' by Andre Agassi

**THE CAUCUS**  
**Senate Ethics Panel Admonishes Burr**  
The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

**MEDIA DECODER**  
**A Tearful Winfrey Explains Her Departure**  
By BRIAN STELTER  
12:18 PM ET  
Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.  
Video: Winfrey's Announcement  
Winfrey Bets on Cable | Times Topics

**OPINION**  
**Introducing a New Opinionator**  
Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gail Collins and David Brooks.

**WEEKEND ARTS**  
**ART REVIEW**  
**Tim Burton at the MoMA**  
The retrospective of his noncinematic art is a let-down, Ken Johnson writes.

**MUSIC REVIEW**  
**'Kepler'**  
Philip Glass examines the relationship between science and religion in his new opera.

**FALL OPEN HOUSE**  
UNIVERSITY OF MICHIGAN  
NOVEMBER 21

**MARKETS** As 2:22 PM ET  
S&P 500 Dow Nasdaq  
1,899.48 18,311.66 2,164.05  
-4.50 -20.7% +12.77  
-0.30% -0.20% -0.65%

**GET QUOTES** My Portfolio [button]  
Stock, ETFs, Funds [button]

**REAL ESTATE** AUTOS JOBS ALL CLASSIFIEDS  
**STREETSCAPES** FIND PROPERTIES  
**A Walk Among Vanished** - Go to Real Estate Section

Ad-mix

Profit maximization

- ad pool
- user information
- ad/user performance

pricing model...

1. cost per mille (CPM)

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

**The New York Times**  
Friday, November 20, 2009 Last Update: 2:01 PM ET

TAKE YOUR OWN PATH

SEARCH [ ]

TRIAL OFFER: TAKE MY OWN PATH. I CHOOSE DELL. LEARN MORE

Switch to Global Edition

**JOBS**  
REAL ESTATE  
AUTOS  
ALL CLASSIFIEDS

**WORLD**  
U.S.  
POLITICS  
N.Y./REGION  
BUSINESS  
TECHNOLOGY

**SPORTS**  
SCIENCE  
HEALTH  
OPINION

**ARTS**  
Books  
Movies  
Music  
Television  
Theater  
STYLE  
Dining & Wine  
Fashion & Style  
Home & Garden  
Weddings/  
Celebrations

**TRAVEL**  
Blogs  
Cartoons /  
Humor  
Classifieds  
Connections  
Crossword /  
Games  
Education

**New Sanctions Considered as Iran Stalls on Nuclear Deal**  
By STEVEN ERLANGER  
7 minutes ago  
Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.  
Times Topics: Iran | Iran's Nuclear Program

**Quiet Shift in Budget Rules May Prompt City Job Cuts**  
By MICHAEL BARBARO  
12:00 PM ET  
The Bloomberg administration, which has said it wants to avert layoffs, has not disclosed the change to the public.

**ROOM FOR DEBATE Reid and the Power of Persuasion**  
By THE EDITORS  
11:10 AM ET  
With a key health care vote coming up, experts discuss the

**A Conversation With Andre Agassi**  
The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.  
Review: 'Open: An Autobiography,' by Andre Agassi

**Senate Ethics Panel Admonishes Burr**  
The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

**MEDIA DECODER: A Tearful Winfrey Explains Her Departure**  
By BRIAN STELTER  
12:18 PM ET  
Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.  
Video: Winfrey's Announcement  
Winfrey Bets on Cable | Times Topics

**OPINION**  
Introducing a New Opinionator  
Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gail Collins and David Brooks.

**WEEKEND ARTS**  
ART REVIEW  
Tim Burton at the MoMA  
The retrospective of his noncinematic art is a let-down, Ken Johnson writes.

**MUSIC REVIEW**  
'Kepler'  
Philip Glass examines the relationship between science and religion in his new opera.

**FALL OPEN HOUSE**  
MONTRELL UNIVERSITY  
NOVEMBER 21

**Krugman: The Big Squads Set** | **Corsets** (4/4)  
**Brooks: A Gethner Win**  
**Editorial: Mammograms**  
**Ross Douthat Blog**  
**Bloggingheads: Afghanistan Strategy**

**ENTER THE 1ST VIRTUAL HOUSTON STORE**  
thevenueavenue.com

**MARKETS** | 11:22 PM ET  
S&P 500 Dow Nasdaq  
1,899.48 18,311.66 2,164.05  
-4.50 -20.7% +12.77  
-0.30% -0.20% -0.60%

**GET QUOTES** | My Portfolio  
Stock, ETFs, Funds | Go

**THIS HOLIDAY SEASON REVISIT THE JOY OF FAMILY**  
ROBERT DE NIRO  
EVERYBODY'S FINE  
RENTAL: EVERETT COLLECTION

REAL ESTATE AUTOS JOBS ALL CLASSIFIEDS  
STREETSCAPES FIND PROPERTIES  
A Walk Among Vanished - Go to Real Estate Section

Ad-mix

Profit maximization

- ad pool
- user information
- ad/user performance

pricing model...

1. cost per mille (CPM)
2. cost per click (CPC)

# Customization in Online Advertisement

Home Page | Today's Paper | Video | Most Popular | Times Topics | Most Recent | Get Home Delivery | Log In | Register Now

**The New York Times**  
Friday, November 20, 2009 Last Update: 2:01 PM ET

TAKE YOUR OWN PATH

SEARCH [input] [button] Try Our EXTRA Home Page [button] Get Home Delivery | Personalize Your Viesiter [button]

BUSINESS **I TAKE MY OWN PATH. I CHOOSE DELL.** LEARN MORE [button]

Switch to Global Edition •

**JOBS**  
REAL ESTATE  
AUTOS  
ALL CLASSIFIEDS

**WORLD**  
U.S.  
POLITICS  
N.Y./REGION  
BUSINESS  
TECHNOLOGY  
SPORTS  
SCIENCE  
HEALTH  
OPINION  
ARTS  
Books  
Movies  
Music  
Television  
THEATER  
Dining & Wine  
Fashion & Style  
Home & Garden  
Weddings/  
Celebrations  
TRAVEL  
Blogs  
Cartoons /  
Humor  
Classifieds  
Connections  
Crossword /  
Games  
Education

**New Sanctions Considered as Iran Stalls on Nuclear Deal**  
By STEVEN ERLANGER  
7 minutes ago  
Officials from six countries and the European Union expressed disappointment that Iran had not accepted a deal to export most of its enriched uranium.  
Times Topics: Iran | Iran's Nuclear Program

**Quiet Shift in Budget Rules May Prompt City Job Cuts**  
By MICHAEL BARBARO  
12:00 PM ET  
The Bloomberg administration, which has said it wants to avert layoffs, has not disclosed the change to the public.

**ROOM FOR DEBATE Reid and the Power of Persuasion**  
By THE EDITORS  
11:18 AM ET  
With a key health care vote coming up, experts discuss the

**OPINION**  
Introducing a New Opinionator  
Columns from Linda Greenhouse and Timothy Egan, and The Conversation, with Gail Collins and David Brooks.

**WEEKEND ARTS**  
ART REVIEW  
**Tim Burton at the MoMA**  
The retrospective of his noncinematic art is a let-down, Ken Johnson writes.

**MUSIC REVIEW**  
**'Kepler'**  
Philip Glass examines the relationship between science and religion in his new opera.

**FALL OPEN HOUSE**  
HINSHILLA UNIVERSITY  
NOVEMBER 21

**MARKETS** 11:22 PM ET  
S&P 500 Dow Nasdaq  
1,899.48 18,311.65 2,164.05  
-4.50 -20.7% +12.77  
-0.30% -0.20% -0.60%

**GET QUOTES** My Portfolio [button]  
Stock, ETFs, Funds [button]

**VIDEO**  
**A Conversation With Andre Agassi**  
The tennis great talked candidly about life on and off the court with Sam Tanenhaus, the editor of the Book Review.  
Review: 'Open: An Autobiography,' by Andre Agassi

**THE CAUCUS**  
**Senate Ethics Panel Admonishes Burr**  
The panel rebuked but did not recommend punishing Senator Roland Burr for "misleading" statements.

**MEDIA DECODER**  
**A Tearful Winfrey Explains Her Departure**  
By BRIAN STELTER  
12:18 PM ET  
Oprah Winfrey confirmed to viewers what she told her staff members on Thursday: She is ending her talk show in 2011.  
Video: Winfrey's Announcement  
Winfrey Bets on Cable | Times Topics

**REAL ESTATE** AUTOS JOBS ALL CLASSIFIEDS  
STREETSCAPES FIND PROPERTIES  
A Walk Among Vanished - Go to Real Estate Section

Ad-mix

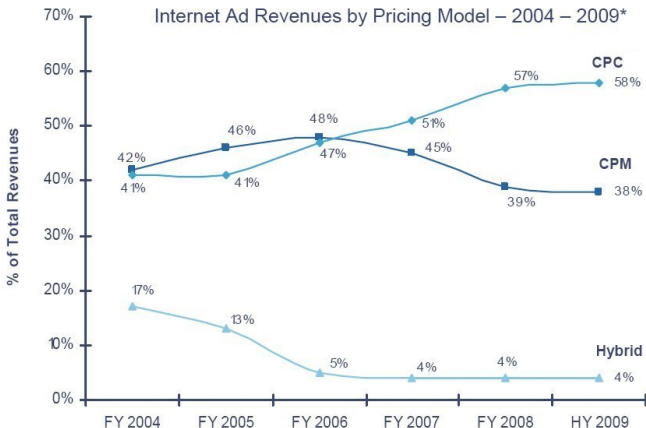
Profit maximization

- ad pool
- user information
- ad/user performance

pricing model...

1. cost per mille (CPM)
2. cost per click (CPC)

# Online Advertisement: Pricing Models



source: Interactive Advertisement Bureau Internet Advertisement Revenue Report (by PricewaterhouseCoopers)



# Customization in Online Advertisement

---

- anonymity on the Internet



July 1993, *The New Yorker*

# Customization in Online Advertisement

---

- anonymity on the Internet
- common practice
  - ▶ internet cookies
  - ▶ list of categories of interest
  - ▶ adaptive to “behavior”



July 1993, *The New Yorker*

# Customization in Online Advertisement

---

- anonymity on the Internet
- common practice
  - ▶ internet cookies
  - ▶ list of categories of interest
  - ▶ adaptive to “behavior”

id=c51781920000060|t=1253117509|et=730



July 1993, *The New Yorker*

# Customization in Online Advertisement

---

- anonymity on the Internet
- common practice
  - ▶ internet cookies
  - ▶ list of categories of interest
  - ▶ adaptive to “behavior”

dog food          frisbees          squirrels

```
id=c51781920000060|t=1253117509|et=730
```



*"On the Internet, nobody knows you're a dog."*

July 1993, *The New Yorker*

# Customization in Online Advertisement

---

- anonymity on the Internet
- common practice
  - ▶ internet cookies
  - ▶ list of categories of interest
  - ▶ adaptive to “behavior”

dog food          frisbees          squirrels

```
id=c51781920000060||t=1253117509|et=730
```

- user information
  - ▶ behavioral, geographical, demographical data ...



July 1993, *The New Yorker*

## Online Advertisement: Salient Features

---

Advent of the Internet has transformed consumer experience of advertisements and media...

### dynamic/customized advertisement display

- one-to-one interaction with users...
- contracts (CPC) are increasingly performance-based
- customization to individual users exploiting side information
- dynamic decision making to balance learning and profits

# Road Map

---

**Focus:** Publisher's display decision in dynamic environment

- I. Customization in online advertisement
  - ▶ publisher's problem definition
  - ▶ need for *dynamic learning* of ad performance
  
- II. Stylized model for display-based online advertisement
  - ▶ limit of achievable performance
  - ▶ policy construction and guarantees
  
- III. Insights and takeaway messages

## Towards a Problem Formulation...

---

**Publisher's decision:** ad/user performance

1. direct revenue: cost per click (cpc)
2. click probability:

user profile + ad mix  $\rightarrow$  click probability



## Towards a Problem Formulation...

---

**Publisher's decision:** ad/user performance

1. direct revenue: cost per click (cpc)
2. click probability:

user profile + ad mix  $\rightarrow$  click probability

- use historical data on profiles, display and click

## Towards a Problem Formulation...

---

**Publisher's decision:** ad/user performance

1. direct revenue: cost per click (cpc)
2. click probability:

user profile + ad mix  $\rightarrow$  click probability

- use historical data on profiles, display and click
- natural approach... fit a choice model

$$\mathbb{P} \{ \text{user clicks on ad} \} = f(\text{ad}, \text{user profile}, \text{ad mix}, \beta)$$

model parameters 

## Towards a Problem Formulation...

---

**Publisher's decision:** ad/user performance

1. direct revenue: cost per click (cpc)
2. click probability:

user profile + ad mix  $\rightarrow$  click probability

- use historical data on profiles, display and click
- natural approach... fit a choice model

$$\mathbb{P}\{\text{user clicks on ad}\} = f(\text{ad}, \text{user profile}, \text{ad mix}, \beta)$$

model parameters 

- additional considerations: display capacity...

## Towards a Problem Formulation...

---

**Publisher's objective:** ideally...

maximize expected revenue from interaction with users

$$\max_{\text{ad mix}} \left\{ \sum_{\text{ad in mix}} \text{cpc}(\text{ad}) \cdot f(\text{ad}, \text{user profile}, \text{ad mix}, \beta) \right\}$$

## Towards a Problem Formulation...

---

**Publisher's objective:** ideally...

maximize expected revenue from interaction with users

$$\max_{\text{ad mix}} \left\{ \sum_{\text{ad in mix}} \text{cpc}(\text{ad}) \cdot f(\text{ad}, \text{user profile}, \text{ad mix}, \beta) \right\}$$

... **dynamic environment**

## Towards a Problem Formulation...

---

**Publisher's objective:** ideally...

maximize expected revenue from interaction with users

$$\max_{\text{ad mix}} \left\{ \sum_{\text{ad in mix}} \text{cpc}(ad) \cdot f(\text{ad}, \text{user profile}, \text{ad mix}, \beta) \right\}$$

... **dynamic environment**

- new contracts: limited or no history of past interaction...

## Towards a Problem Formulation...

---

**Publisher's objective:** ideally...

maximize expected revenue from interaction with users

$$\max_{\text{ad mix}} \left\{ \sum_{\text{ad in mix}} \text{cpc}(\text{ad}) \cdot f(\text{ad}, \text{user profile}, \text{ad mix}, \beta) \right\}$$

... **dynamic environment**

- new contracts: limited or no history of past interaction...
- contract expiration ...

## Towards a Problem Formulation...

---

**Publisher's objective:** ideally...

maximize expected revenue from interaction with users

$$\max_{\text{ad mix}} \left\{ \sum_{\text{ad in mix}} \text{cpc}(\text{ad}) \cdot f(\text{ad}, \text{user profile}, \text{ad mix}, \beta) \right\}$$

... **dynamic environment**

- new contracts: limited or no history of past interaction...
- contract expiration ...

estimation accuracy vs profit maximization



## Related Literature

---

### Learning approach to interactive marketing

- Gooley and Lattin (2000)
  - ▶ message customization
- Bertsimas and Mersereau (2007)
  - ▶ solve for each segment separately

## Related Literature

---

### Learning approach to interactive marketing

- Gooley and Lattin (2000)
  - ▶ message customization
- Bertsimas and Mersereau (2007)
  - ▶ solve for each segment separately

### Multi Armed Bandit (MAB) Literature

- Slivkins (2009), Lu et al (2009)
  - ▶ side information: MAB in metric spaces

# Roadmap

---

I. Customization in online advertisement

II. Stylized model for display-based online advertisement

III. Insights and takeaway messages

## Problem Formulation

---

### Stylized model for display-based online advertisement

- finite users ( $T$ ) arrive sequentially
- finite pool of ads ( $\mathcal{N}$ ) with given profit margins ( $w_i$ )
- ad-mix ( $s \in \mathcal{S}$ )...

## Problem Formulation

---

### Stylized model for display-based online advertisement

- finite users ( $T$ ) arrive sequentially
  - finite pool of ads ( $\mathcal{N}$ ) with given profit margins  $\overbrace{(w_i)}^{\text{CPC}}$
  - ad-mix ( $s \in \mathcal{S}$ )...
- ad index

## Problem Formulation

---

### Stylized model for display-based online advertisement

- finite users ( $T$ ) arrive sequentially
- finite pool of ads ( $\mathcal{N}$ ) with given profit margins  $\overbrace{(w_i)}^{\text{CPC}}$   
ad index
- ad-mix ( $s \in \mathcal{S}$ )...  
display capacity,  $|s| \leq C$

## Problem Formulation

---

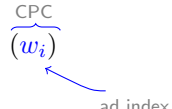

### Stylized model for display-based online advertisement

- finite users ( $T$ ) arrive sequentially
  - finite pool of ads ( $\mathcal{N}$ ) with given profit margins  $\overbrace{(w_i)}^{\text{CPC}}$   
ad index
  - ad-mix ( $s \in \mathcal{S}$ )...  
display capacity,  $|s| \leq C$
- ▶ ad-slots are interchangeable, no budget constraints

## Problem Formulation

---

### Stylized model for display-based online advertisement

- finite users ( $T$ ) arrive sequentially
  - finite pool of ads ( $\mathcal{N}$ ) with given profit margins  $\overbrace{(w_i)}^{\text{CPC}}$   

  - **ad-mix** ( $s \in \mathcal{S}$ )...  

- ▶ ad-slots are interchangeable, no budget constraints

- **objective:** maximize revenue by suitable ad display policy
- user clicks on at most one ad ...
- users are utility maximizers

**U**(user profile, ad) + **ad-mix** → **click decision**



## Problem Formulation: User Utilities

---

### Logit model with **user-specific** mean utility

$$U_i = \beta_i \cdot x + \epsilon_i$$

utility of ad  $i$   $\swarrow$   $\nwarrow$  noise (Gumbel)  
ad factors  $\nearrow$   $\searrow$  user profile

- user profile  $x$  is  $d$ -dimensional vector [observed]
- ad factors  $\beta_i$  is  $d$ -dimensional vector [to be estimated]

## Problem Formulation: User Utilities

---

### Logit model with **user-specific** mean utility

$$U_i = \beta_i \cdot x + \epsilon_i$$

utility of ad  $i$   $\swarrow$   $\nwarrow$  noise (Gumbel)  
ad factors  $\nearrow$   $\searrow$  user profile (unobserved...)

- user profile  $x$  is  $d$ -dimensional vector [observed]
- ad factors  $\beta_i$  is  $d$ -dimensional vector [to be estimated]

## Problem Formulation: User Utilities

---

### Logit model with **user-specific** mean utility

$$U_i = \beta_i \cdot x + \epsilon_i$$

utility of ad  $i$       noise (Gumbel)

ad factors      user profile

- user profile  $x$  is  $d$ -dimensional vector [observed]
- ad factors  $\beta_i$  is  $d$ -dimensional vector [to be estimated]

# Problem Formulation: User Utilities

---

## Logit model with **user-specific** mean utility

$$U_i = \beta_i \cdot x + \epsilon_i$$

utility of ad  $i$       noise (Gumbel)

ad factors      user profile

- user profile  $x$  is  $d$ -dimensional vector [observed]
- ad factors  $\beta_i$  is  $d$ -dimensional vector [to be estimated]

$x$

$$\begin{pmatrix} 0.4 \\ 0.9 \\ 35 \\ 1 \end{pmatrix} \quad \begin{array}{l} \text{sport affinity} \\ \text{prob. male} \\ \text{exp. age} \\ \text{dummy} \end{array}$$

## Problem Formulation: User Utilities

---

### Logit model with **user-specific** mean utility

$$U_i = \beta_i \cdot x + \epsilon_i$$

utility of ad  $i$   $\rightarrow$   $U_i$   $\leftarrow$  noise (Gumbel)  
ad factors  $\rightarrow$   $\beta_i$   $\leftarrow$  user profile  $x$

- user profile  $x$  is  $d$ -dimensional vector [observed]
- ad factors  $\beta_i$  is  $d$ -dimensional vector [to be estimated]

$$\text{running shoes } \left( \begin{array}{c} \beta_i \\ 0.3 \\ 1.9 \\ -0.3 \\ 2.3 \end{array} \right)^T \left( \begin{array}{c} x \\ 0.4 \\ 0.9 \\ 35 \\ 1 \end{array} \right) \begin{array}{l} \text{sport affinity} \\ \text{prob. male} \\ \text{exp. age} \\ \text{dummy} \end{array}$$

## Problem Formulation: User Utilities

---

### Logit model with **user-specific** mean utility

$$U_i = \beta_i \cdot x + \epsilon_i$$

utility of ad  $i$   $\swarrow$   $\searrow$  noise (Gumbel)  
ad factors  $\nearrow$   $\nwarrow$  user profile

- user profile  $x$  is  $d$ -dimensional vector [observed]
- ad factors  $\beta_i$  is  $d$ -dimensional vector [to be estimated]

### our approach: Logistic regression (profiles)

$$f_i(s, x, \beta) = \frac{\exp\{\beta_i \cdot x\}}{1 + \sum_{j \in s} \exp\{\beta_j \cdot x\}}$$



ad mix  $\nearrow$

## Problem Formulation

---

- expected revenue from displaying ad mix  $s$  to user profile  $x$ :

$$r(s, x, \beta) = \sum_{i \in s} w_i \cdot \left( \frac{\exp\{\beta_i \cdot x\}}{1 + \sum_{j \in s} \exp\{\beta_j \cdot x\}} \right)$$



ad profit margin   logit click prob.

## Problem Formulation

---

- expected revenue from displaying **ad mix**  $s$  to **user profile**  $x$ :

$$r(s, x, \beta) = \sum_{i \in s} w_i \cdot \left( \frac{\exp\{\beta_i \cdot x\}}{1 + \sum_{j \in s} \exp\{\beta_j \cdot x\}} \right)$$

ad profit margin   logit click prob.

- **profile**  $X_t$  drawn from a **finite** set  $\mathcal{X}$  according to distribution  $G$ 
  - ▶ finite number of user segments. . .
  - ▶  $G$  reflects histogram of population





## Problem Formulation

---

- expected revenue from displaying **ad mix**  $s$  to **user profile**  $x$ :

$$r(s, x, \beta) = \sum_{i \in s} w_i \cdot \left( \frac{\exp\{\beta_i \cdot x\}}{1 + \sum_{j \in s} \exp\{\beta_j \cdot x\}} \right)$$

ad profit margin   logit click prob.

- **profile**  $X_t$  drawn from a **finite** set  $\mathcal{X}$  according to distribution  $G$ 
  - ▶ finite number of user segments. . .
  - ▶  $G$  reflects histogram of population

**ad**  $i$  factors  $\beta_i$  **initially unknown** for all ads




# Oracle Benchmark

---

Suppose publisher knows  $\beta$  a priori

- formulate and solve an optimization problem

$$J^*(T|\beta) := \sup_{s(\cdot)} \mathbb{E} \left[ \sum_{t=1}^T r(s(t), X_t, \beta) \right]$$

known parameters   

feasible ad policies expected revenue

## Oracle Benchmark

---

Suppose publisher knows  $\beta$  a priori

- formulate and solve an optimization problem

$$J^*(T|\beta) := \sup_{s(\cdot)} \mathbb{E} \left[ \sum_{t=1}^T r(s(t), X_t, \beta) \right]$$

known parameters  $\uparrow$   $s(\cdot)$   $\uparrow$  feasible ad policies  $\uparrow$  expected revenue

**Oracle policy:** offer  $s^*(X_t, \beta)$  to user  $t$

$$s^*(x, \beta) \in \operatorname{argmax} \{ r(s, x, \beta) : s \in \mathcal{S} \}$$


expected revenue  $\uparrow$

## Measuring Policy Performance

---

- **ad mix** decision for feasible policies based on history of past interaction and current **user profile**
- performance of ad mix policy  $\pi$ :  
**revenue loss** relative to oracle policy

$$\mathcal{R}(\pi, T) := J^*(T|\beta) - \mathbb{E} \left[ \sum_{t=1}^T r(\mathbf{s}^\pi(t), \mathbf{X}_t, \beta) \right]$$


expected revenue 

## Measuring Policy Performance

---

- **ad mix** decision for feasible policies based on history of past interaction and current **user profile**
- performance of ad mix policy  $\pi$ :  
**revenue loss** relative to oracle policy

$$\mathcal{R}(\pi, T) := J^*(T|\beta) - \mathbb{E} \left[ \sum_{t=1}^T r(\mathbf{s}^\pi(t), \mathbf{X}_t, \beta) \right]$$

expected revenue 

**Main Q:** **how small** can we make this revenue loss?

structure of an **optimal policy**?

## Limit of Achievable Performance

---

Theorem [ Saure and Z (2012) ]

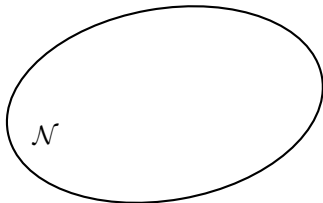
Any *good* policy  $\pi$  must incur revenue loss

$$\mathcal{R}(\pi, T) \geq \sum_{i \in \mathcal{N}} K_i \log T$$

## Limit of Achievable Performance

---

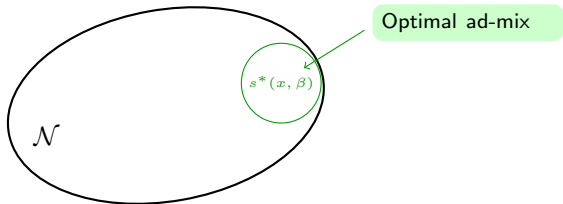
- Fix user profile  $x \in \mathcal{X}$ :



## Limit of Achievable Performance

---

- Fix user profile  $x \in \mathcal{X}$ :

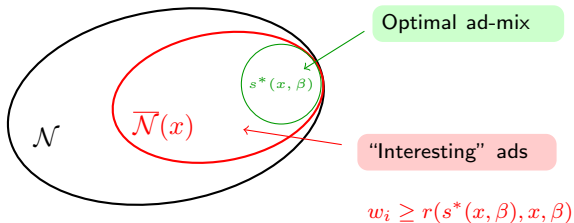




## Limit of Achievable Performance

---

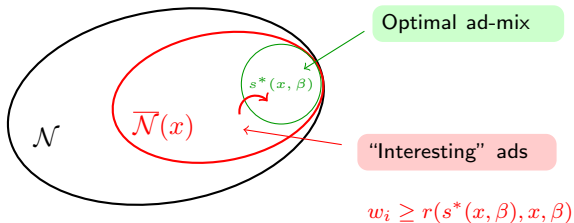
- Fix user profile  $x \in \mathcal{X}$ :



## Limit of Achievable Performance

---

- Fix user profile  $x \in \mathcal{X}$ :



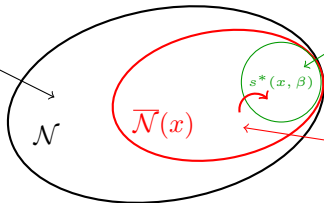
## Limit of Achievable Performance

---

- Fix user profile  $x \in \mathcal{X}$ :

“Uninteresting” ads

$$w_i < r(s^*(x, \beta), x, \beta)$$



Optimal ad-mix

“Interesting” ads

$$w_i \geq r(s^*(x, \beta), x, \beta)$$

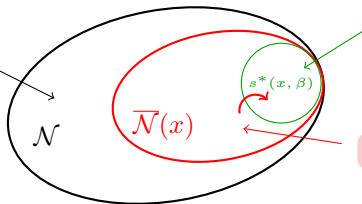
## Limit of Achievable Performance

---

- Fix user profile  $x \in \mathcal{X}$ :

“Uninteresting” ads

$$w_i < r(s^*(x, \beta), x, \beta)$$

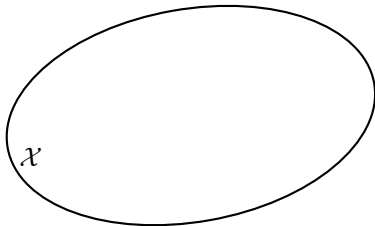


Optimal ad-mix

“Interesting” ads

$$w_i \geq r(s^*(x, \beta), x, \beta)$$

- Fix ad  $i \in \mathcal{N}$ :

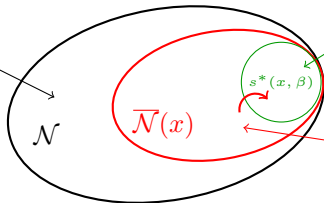


## Limit of Achievable Performance

- Fix user profile  $x \in \mathcal{X}$ :

“Uninteresting” ads

$$w_i < r(s^*(x, \beta), x, \beta)$$

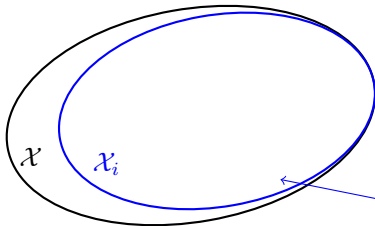


Optimal ad-mix

“Interesting” ads

$$w_i \geq r(s^*(x, \beta), x, \beta)$$

- Fix ad  $i \in \mathcal{N}$ :



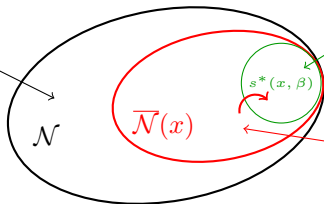
$i$  is “Interesting”

## Limit of Achievable Performance

- Fix user profile  $x \in \mathcal{X}$ :

“Uninteresting” ads

$$w_i < r(s^*(x, \beta), x, \beta)$$

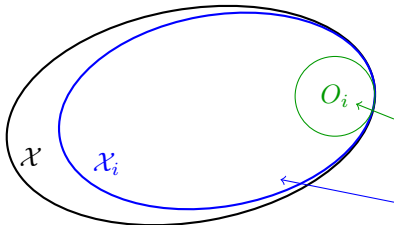


Optimal ad-mix

“Interesting” ads

$$w_i \geq r(s^*(x, \beta), x, \beta)$$

- Fix ad  $i \in \mathcal{N}$ :



$i$  is optimal

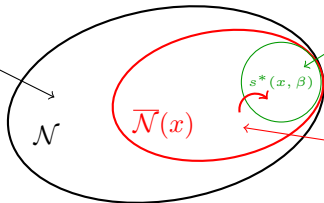
$i$  is “Interesting”

## Limit of Achievable Performance

- Fix user profile  $x \in \mathcal{X}$ :

“Uninteresting” ads

$$w_i < r(s^*(x, \beta), x, \beta)$$

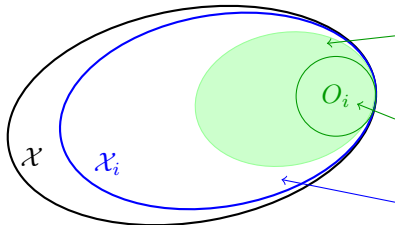


Optimal ad-mix

“Interesting” ads

$$w_i \geq r(s^*(x, \beta), x, \beta)$$

- Fix ad  $i \in \mathcal{N}$ :



$\text{span}(O_i)$

$i$  is optimal

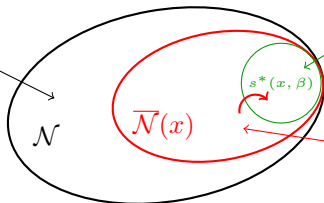
$i$  is “Interesting”

## Limit of Achievable Performance

- Fix user profile  $x \in \mathcal{X}$ :

“Uninteresting” ads

$$w_i < r(s^*(x, \beta), x, \beta)$$



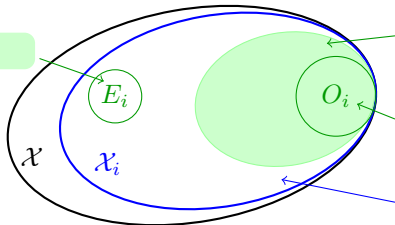
Optimal ad-mix

“Interesting” ads

$$w_i \geq r(s^*(x, \beta), x, \beta)$$

- Fix ad  $i \in \mathcal{N}$ :

additional exploration



$span(O_i)$

$i$  is optimal

$i$  is “Interesting”

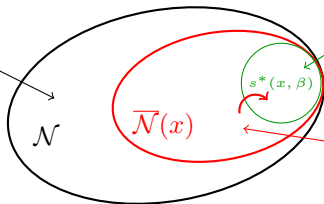


## Limit of Achievable Performance

- Fix user profile  $x \in \mathcal{X}$ :

“Uninteresting” ads

$$w_i < r(s^*(x, \beta), x, \beta)$$



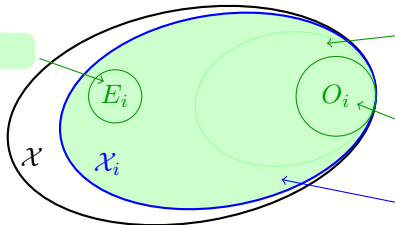
Optimal ad-mix

“Interesting” ads

$$w_i \geq r(s^*(x, \beta), x, \beta)$$

- Fix ad  $i \in \mathcal{N}$ :

additional exploration



$span(O_i)$

$i$  is optimal

$i$  is “Interesting”

## Limit of Achievable Performance

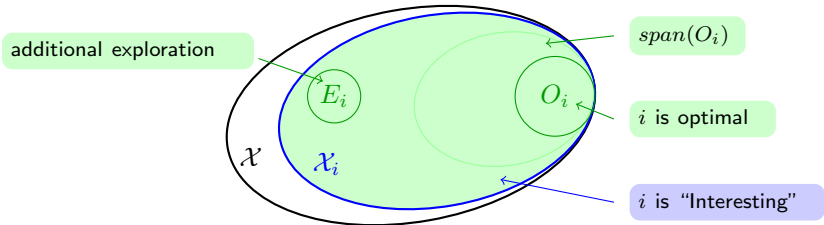
---

Theorem [ Saure and Z (2012) ]

Any *good* policy  $\pi$  must incur revenue loss

$$\mathcal{R}(\pi, T) \geq \sum_{i \in \mathcal{N}} K_i \log T$$

- Fix ad  $i \in \mathcal{N}$  :



## Limit of Achievable Performance

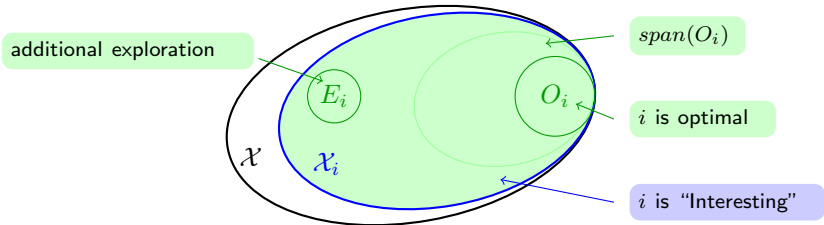
---

Theorem [ Saure and Z (2012) ]  $K_i \sim \text{rank}(\mathcal{X}_i) - \text{rank}(O_i)$

Any good policy  $\pi$  must incur revenue loss

$$\mathcal{R}(\pi, T) \geq \sum_{i \in \mathcal{N}} K_i \log T$$

- Fix ad  $i \in \mathcal{N}$  :



## Qualitative Insights and Policy Design

---

**Ad/profile exploration as source of revenue loss**

# Qualitative Insights and Policy Design

---

## Ad/profile exploration as source of revenue loss

- for a given ad, there is **no need** to estimate mean utilities for **every** profile
  - ▶ need to assess performance only on some profiles ( $\mathcal{X}_i$ )
  - ▶ use information on set **spanning** such profiles

# Qualitative Insights and Policy Design

---

## Ad/profile exploration as source of revenue loss

- for a given ad, there is **no need** to estimate mean utilities for **every** profile
  - ▶ need to assess performance only on some profiles ( $\mathcal{X}_i$ )
  - ▶ use information on set **spanning** such profiles
- use information that **does not** contribute to revenue loss
  - ▶ use profiles for which an ad is optimal

# Qualitative Insights and Policy Design

---

## Ad/profile exploration as source of revenue loss

- for a given ad, there is **no need** to estimate mean utilities for **every** profile
  - ▶ need to assess performance only on some profiles ( $\mathcal{X}_i$ )
  - ▶ use information on set **spanning** such profiles
- use information that **does not** contribute to revenue loss
  - ▶ use profiles for which an ad is optimal
- information contributing to revenue loss **must be capped**
  - ▶ performed on **order  $\log T$**  users. . .

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )



## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )

**Construction:**

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )

**Construction:**

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\frac{\mathbb{E} \# \text{ clicks on ad } i \text{ for profile } x}{\mathbb{E} \# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\beta_i \cdot x)$$

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )

**Construction:**

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} \approx \exp(\beta_i \cdot x)$$

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )

**Construction:**

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} \approx \exp(\beta_i \cdot x), x \in E_i$$

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )

**Construction:**

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\hat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x), x \in E_i \right\}$$

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )  
⏟  
spanning  $\mathcal{X}_i$ ,

**Construction:**

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\hat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x), x \in E_i \right\}$$

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )  
⏟  
spanning  $\mathcal{X}_i$ ,

### Construction:

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\hat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x), x \in E_i \right\}$$

- adapt  $E_i$  to span a proxy for  $\mathcal{X}_i \dots$

## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )  
spanning  $\mathcal{X}_i$ , including  $O_i$

### Construction:

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\hat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x), x \in E_i \right\}$$

- adapt  $E_i$  to span a proxy for  $\mathcal{X}_i \dots$



## Policy design: Key ingredients

---

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )  
spanning  $\mathcal{X}_i$ , including  $O_i$

### Construction:

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\hat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x), x \in E_i \right\}$$

- adapt  $E_i$  to span a proxy for  $\mathcal{X}_i \dots$  use most explored profiles

## Policy design: Key ingredients

---

order  $\log T$  users

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )  
spanning  $\mathcal{X}_i$ , including  $O_i$

### Construction:

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\hat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x), x \in E_i \right\}$$

- adapt  $E_i$  to span a proxy for  $\mathcal{X}_i \dots$  use most explored profiles

## Policy design: Key ingredients

---

order  $\log T$  users

**Intuition:** force **right frequency** of ad  $i$  experimentation  
on **suitable** estimation-set ( $E_i \in \mathcal{X}$ )  
spanning  $\mathcal{X}_i$ , including  $O_i$

### Construction:

- estimate model parameter for ad  $i$  using only information on profiles in  $E_i$

$$\hat{\beta}_i \in \left\{ \rho \in \mathbb{R}^d : \frac{\# \text{ clicks on ad } i \text{ for profile } x}{\# \text{ no clicks and ad } i \text{ offered for profile } x} = \exp(\rho \cdot x), x \in E_i \right\}$$

- adapt  $E_i$  to span a proxy for  $\mathcal{X}_i$  ... use most explored profiles
- for user  $t$  force **order- $(\log t)$**  exploration on  $E_i$

## Structure of the Proposed Policy

---

**Algorithm structure:**  $\pi^* = \pi(\kappa) \leftarrow$  tuning parameter

## Structure of the Proposed Policy

---

**Algorithm structure:**  $\pi^* = \pi(\kappa) \leftarrow$  tuning parameter

- Initialize exploration sets  $E_i$  for all ad  $i$

## Structure of the Proposed Policy

---

**Algorithm structure:**  $\pi^* = \pi(\kappa) \leftarrow$  tuning parameter

- Initialize exploration sets  $E_i$  for all ad  $i$
- for every user  $t$ :

## Structure of the Proposed Policy

---

**Algorithm structure:**  $\pi^* = \pi(\kappa) \leftarrow$  tuning parameter

- Initialize exploration sets  $E_i$  for all ad  $i$
- for every user  $t$ :
  - ▶ for ad  $i$ : get estimate for  $\beta_i$  using exploration set  $E_i$

## Structure of the Proposed Policy

---

**Algorithm structure:**  $\pi^* = \pi(\kappa)$  ← tuning parameter

- Initialize exploration sets  $E_i$  for all ad  $i$
- for every user  $t$ :
  - ▶ for ad  $i$ : get estimate for  $\beta_i$  using exploration set  $E_i$
  - ▶ for ad  $i$ : use  $\hat{\beta}$  to update  $E_i$  to span  $\hat{\mathcal{X}}_i$  (most explored)



## Structure of the Proposed Policy

---

**Algorithm structure:**  $\pi^* = \pi(\kappa) \leftarrow$  tuning parameter

- Initialize exploration sets  $E_i$  for all ad  $i$
- for every user  $t$ :
  - ▶ for ad  $i$ : get estimate for  $\beta_i$  using exploration set  $E_i$
  - ▶ for ad  $i$ : use  $\hat{\beta}$  to update  $E_i$  to span  $\hat{\mathcal{X}}_i$  (most explored)
  - ▶ **EXPLORE** on ads for which **user  $t$  profile**:
    1. is useful for estimation ( $X_t \in E_i$ )
    2. is under-tested (displayed to  $\leq \kappa \log t$  such users)

## Structure of the Proposed Policy

---

**Algorithm structure:**  $\pi^* = \pi(\kappa) \leftarrow$  tuning parameter

- Initialize exploration sets  $E_i$  for all ad  $i$
- for every user  $t$ :
  - ▶ for ad  $i$ : get estimate for  $\beta_i$  using exploration set  $E_i$
  - ▶ for ad  $i$ : use  $\hat{\beta}$  to update  $E_i$  to span  $\hat{\mathcal{X}}_i$  (most explored)
  - ▶ **EXPLORE** on ads for which **user  $t$  profile**:
    1. is useful for estimation ( $X_t \in E_i$ )
    2. is under-tested (displayed to  $\leq \kappa \log t$  such users)
  - ▶ otherwise, **EXPLOIT** approximate oracle solution  $s^*(X_t, \hat{\beta})$

## Performance of the Proposed Policy

---

Theorem [ Saure and Z (2012) ]

For suitable chosen tuning parameter  $\kappa$ ,

$$\mathcal{R}(\pi^*, T) \leq \bar{K} \sum_{i \in \mathcal{N}} (\text{rank}(\mathcal{X}_i) - \text{rank}(O_i)) \log T + K,$$

where  $\bar{K}, K > 0$  are finite constants

## Performance of the Proposed Policy

---

Theorem [ Saure and Z (2012) ]

For suitable chosen tuning parameter  $\kappa$ ,

$$\mathcal{R}(\pi^*, T) \leq \bar{K} \sum_{i \in \mathcal{N}} (\text{rank}(\mathcal{X}_i) - \text{rank}(O_i)) \log T + K,$$

where  $\bar{K}, K > 0$  are finite constants

policy is essentially optimal

## Performance of the Proposed Policy

---

Theorem [ Saure and Z (2012) ]

For suitable chosen tuning parameter  $\kappa$ ,

$$\mathcal{R}(\pi^*, T) \leq \bar{K} \sum_{i \in \mathcal{N}} (\text{rank}(\mathcal{X}_i) - \text{rank}(O_i)) \log T + K,$$

where  $\bar{K}, K > 0$  are finite constants

policy is essentially optimal

**Key results:** for each profile

- *uninteresting ads* displayed to **finite (independent of  $T$ )** number of users
- *ads in the optimal mix* displayed outside that mix **finitely** many times

## Proof Sketch

---

- discrete nature of optimization problem

min optimality gap  
across profiles

+

continuity of expected  
revenue w.r.t  $\beta$

$\Rightarrow$

threshold on  
estimation error

## Proof Sketch

---

- discrete nature of optimization problem

min optimality gap across profiles + continuity of expected revenue w.r.t  $\beta$   $\Rightarrow$  threshold on estimation error

- parameter estimation with  $O(\log t)$  tests

threshold error  $\curvearrowright$  tuning parameter

$$\mathbb{P} \left\{ \|\beta_i - \hat{\beta}_i\|_\infty > \epsilon \right\} \leq \exp(-c\kappa \log t) = \frac{1}{t^{c\kappa}}$$

## Proof Sketch

---

- discrete nature of optimization problem

min optimality gap across profiles + continuity of expected revenue w.r.t  $\beta$   $\Rightarrow$  threshold on estimation error

- parameter estimation with  $O(\log t)$  tests

threshold error  $\downarrow$  tuning parameter  $\downarrow$

$$\mathbb{P} \left\{ \|\beta_i - \hat{\beta}_i\|_\infty > \epsilon \right\} \leq \exp(-c\kappa \log t) = \frac{1}{t^{c\kappa}}$$

- balance exploration and exploitation error ( $\kappa > c^{-1}$ )

$\leftarrow$   $\leftarrow$

$$\mathcal{R}(\pi^*, T) \leq O \left( \kappa \log T + \sum_{t=1}^T \frac{1}{t^{c\kappa}} \right)$$



## Numerical Illustration

---

- 4 products, 3 two-dimensional profiles
- feasible set  $\mathcal{S} := \{s \subset \mathcal{N} : |s| \leq 2\}$ ,  $\kappa = 40$

$$\beta = \begin{pmatrix} -1.30 & 2.00 & 2.75 & 3.00 \\ 3.00 & 2.00 & 2.75 & -1.30 \end{pmatrix} \quad \mathcal{X} = \begin{pmatrix} 0.1 & 0.5 & 0.9 \\ 0.9 & 0.5 & 0.1 \end{pmatrix}$$

### Oracle solution

profile	$x_1$	$x_2$	$x_3$
opt. mix	{1, 2}	{2, 3}	{2, 4}
opt. revenue	0.587	0.546	0.578
uninteresting	{3}	-	{3}

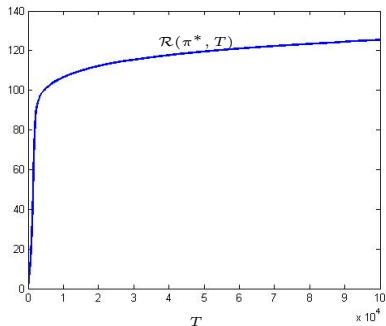
## Numerical Illustration

- 4 products, 3 two-dimensional profiles
- feasible set  $\mathcal{S} := \{s \subset \mathcal{N} : |s| \leq 2\}$ ,  $\kappa = 40$

$$\beta = \begin{pmatrix} -1.30 & 2.00 & 2.75 & 3.00 \\ 3.00 & 2.00 & 2.75 & -1.30 \end{pmatrix} \quad \mathcal{X} = \begin{pmatrix} 0.1 & 0.5 & 0.9 \\ 0.9 & 0.5 & 0.1 \end{pmatrix}$$

### Oracle solution

profile	$x_1$	$x_2$	$x_3$
opt. mix	{1, 2}	{2, 3}	{2, 4}
opt. revenue	0.587	0.546	0.578
uninteresting	{3}	-	{3}



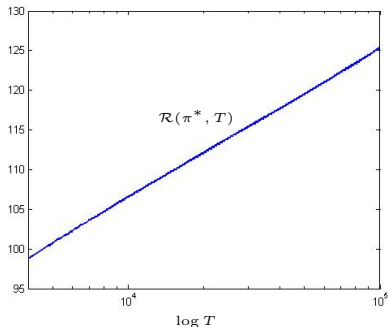
## Numerical Illustration

- 4 products, 3 two-dimensional profiles
- feasible set  $\mathcal{S} := \{s \subset \mathcal{N} : |s| \leq 2\}$ ,  $\kappa = 40$

$$\beta = \begin{pmatrix} -1.30 & 2.00 & 2.75 & 3.00 \\ 3.00 & 2.00 & 2.75 & -1.30 \end{pmatrix} \quad \mathcal{X} = \begin{pmatrix} 0.1 & 0.5 & 0.9 \\ 0.9 & 0.5 & 0.1 \end{pmatrix}$$

### Oracle solution

profile	$x_1$	$x_2$	$x_3$
opt. mix	{1, 2}	{2, 3}	{2, 4}
opt. revenue	0.587	0.546	0.578
uninteresting	{3}	-	{3}



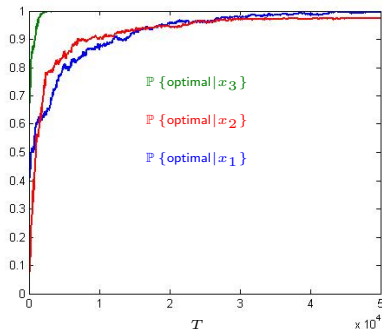
## Numerical Illustration

- 4 products, 3 two-dimensional profiles
- feasible set  $\mathcal{S} := \{s \subset \mathcal{N} : |s| \leq 2\}$ ,  $\kappa = 40$

$$\beta = \begin{pmatrix} -1.30 & 2.00 & 2.75 & 3.00 \\ 3.00 & 2.00 & 2.75 & -1.30 \end{pmatrix} \quad \mathcal{X} = \begin{pmatrix} 0.1 & 0.5 & 0.9 \\ 0.9 & 0.5 & 0.1 \end{pmatrix}$$

### Oracle solution

profile	$x_1$	$x_2$	$x_3$
opt. mix	{1, 2}	{2, 3}	{2, 4}
opt. revenue	0.587	0.546	0.578
uninteresting	{3}	-	{3}



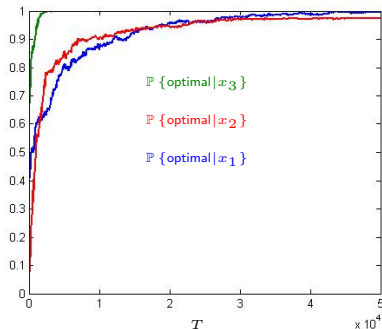
## Numerical Illustration

- 4 products, 3 two-dimensional profiles
- feasible set  $\mathcal{S} := \{s \subset \mathcal{N} : |s| \leq 2\}$ ,  $\kappa = 40$

$$\beta = \begin{pmatrix} -1.30 & 2.00 & 2.75 & 3.00 \\ 3.00 & 2.00 & 2.75 & -1.30 \end{pmatrix} \quad \mathcal{X} = \begin{pmatrix} 0.1 & 0.5 & 0.9 \\ 0.9 & 0.5 & 0.1 \end{pmatrix}$$

### Oracle solution

profile	$x_1$	$x_2$	$x_3$
opt. mix	{1, 2}	{2, 3}	{2, 4}
opt. revenue	0.587	0.546	0.578
uninteresting	{3}	-	{3}
anon. mix	{1, 2}	{1, 2}	{1, 2}
anon. revenue	0.587	0.543	<b>0.525</b>



# Roadmap

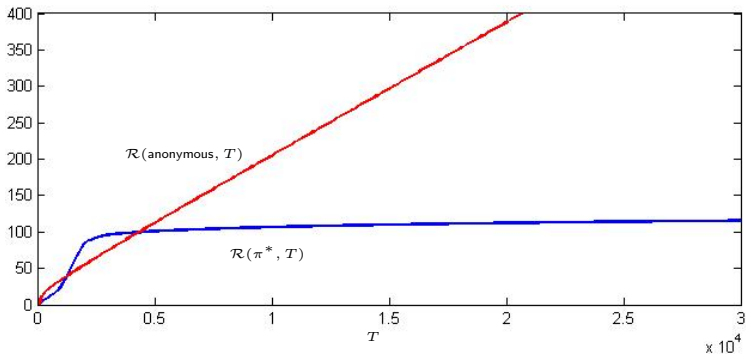
---

- I. Customization in online advertisement
- II. Stylized model for display-based online advertisement
- III. Insights and takeaway messages

## Insights and Takeaway Messages

---

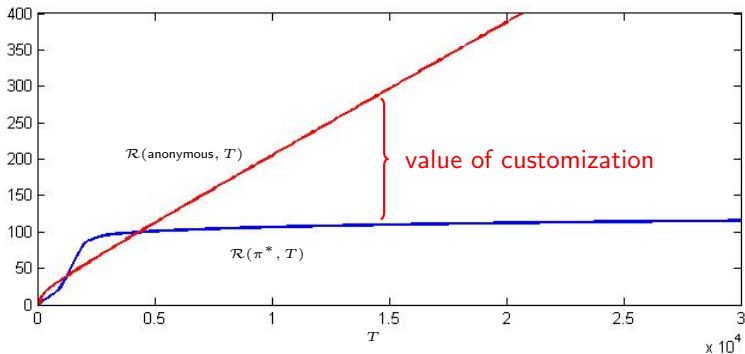
- value of customization
  - ▶ speed of learning
  - ▶ misspecification risk



## Insights and Takeaway Messages

---

- value of customization
  - ▶ speed of learning
  - ▶ misspecification risk





## Insights and Takeaway Messages

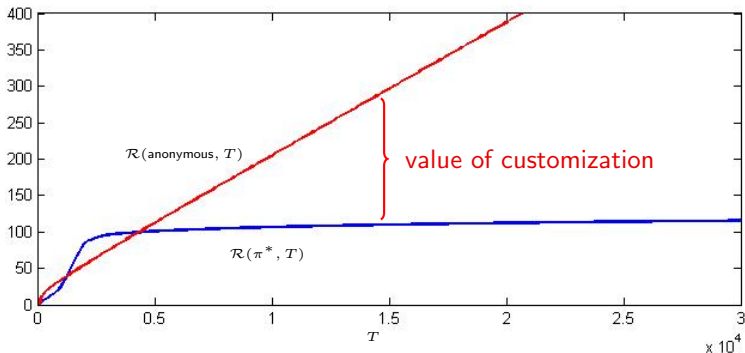
---

### ■ value of customization

- ▶ speed of learning
- ▶ misspecification risk

### ■ cost of information

- ▶ “suboptimal” exploration
- ▶ dependence on structure



## Insights and Takeaway Messages

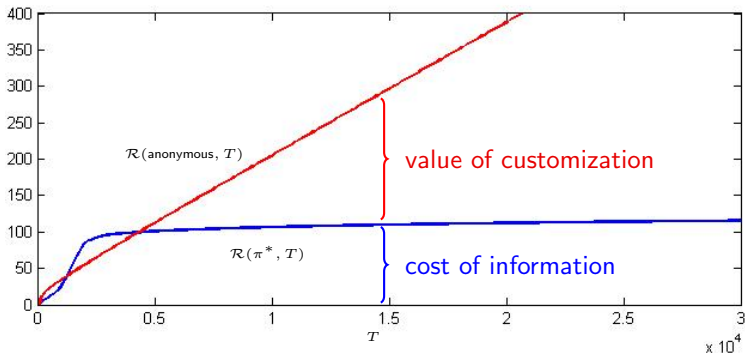
---

### ■ value of customization

- ▶ speed of learning
- ▶ misspecification risk

### ■ cost of information

- ▶ “suboptimal” exploration
- ▶ dependence on structure



## Final Thoughts

---

- concepts
  - ▶ semi-myopic type policies [ avoid incomplete learning ]
  - ▶ minimal exploration needed
  - ▶ significant gains from customizing policies to application

## Final Thoughts

---

- concepts
  - ▶ semi-myopic type policies [ avoid incomplete learning ]
  - ▶ minimal exploration needed
  - ▶ significant gains from customizing policies to application
- analysis tools / machinery
  - ▶ information theoretic inequalities [ lower bounds ]
  - ▶ martingale methods, large deviation bounds [ analysis of policies ]
  - ▶ sequential hypothesis testing

## Final Thoughts

---

- concepts
  - ▶ semi-myopic type policies [ avoid incomplete learning ]
  - ▶ minimal exploration needed
  - ▶ significant gains from customizing policies to application
- analysis tools / machinery
  - ▶ information theoretic inequalities [ lower bounds ]
  - ▶ martingale methods, large deviation bounds [ analysis of policies ]
  - ▶ sequential hypothesis testing
- related recent applications of MAB
  - ▶ dynamic content referral [ Besbes, Gur and Z (2012a) ]
  - ▶ temperature tracking and restless bandits [ Besbes and Z (2012b) ]
  - ▶ personalization (Pandora, various recommendation systems etc)
  - ▶ dynamic design of experiments / screening
  - ▶ cognitive radio [ Lai et al (2011) ]
  - ▶ mechanism design formulation [ Kakade et al (2012) ]